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A CRITICAL REVIEW OF THE WORLD LITERATURE IN APPLIED MECHANICS AND RELATED ENGINEERING SCIENCE

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APPLIED MECHANICS *Reviews*

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APPLIED MECHANICS REVIEWS

VOL. 4, NO. 4 | **MARTIN GOLAND** *Editor* | APRIL 1951

A SURVEY OF DEVELOPMENTS IN RHEOLOGY

MARKUS REINER

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1 WHEN reviewing present-day rheology one does well to remember its starting point about a generation ago. Rheology started when measurements of the "viscosity" (η) of different materials showed that this was not a "constant." The first point to keep in mind is the fact that these measurements were made in apparatus where the material was subjected to laminar flow, i.e., simple shear of cylindrical laminae. It is true that, earlier, Trouton (1906) had determined another coefficient of viscosity—which has become known as the "coefficient of viscous traction" (λ)—in pure extensional flow, where "pure" refers to the absence of rotation, and had attempted to relate λ with η, proposing

$$\lambda = 3\eta \qquad [1]$$

but his investigations received attention only after rheology was well established. We shall come back to this point in section 3. What Bingham and collaborators discovered (1916, 1919) when they examined a variable "apparent viscosity" η', was the fact that some of the materials were actually not liquids but very soft plastic solids. They were concentrated suspensions of solid particles in viscous liquids characterized by *two* material constants, viz., a yield value ϑ and a plastic viscosity η_{pl} and, even if these two were constants, a viscosity η' calculated from the usual formulas would show variability. However, Bingham also found (1929) that there are other "anomalous" materials which do not possess any plastic strength and must accordingly be regarded as liquids. A "cut" was therefore made in the anomalous materials subsuming one part into the class of solids, the other into the class of liquids.[1] The problem of the solids required an extension of the theory of plasticity to measurable rates of flow. The ideal material representing this class has been named Bingham-body. Its rheological equation of state was finally established by Oldroyd [1947; AMR 1, Rev. 93], following an earlier suggestion of Reiner (1943), in the form

$$p_{(0)rs} = 2\left[\eta_{pl} + \frac{\vartheta}{(2f_{(0)\alpha\beta}\,f_{(0)\beta\alpha})^{1/2}}\right]f_{(0)rs} \qquad [2][2]$$

which for vanishing η_{pl} degenerates into the equation of state of the Saint Venant body, the ideal prototype of the theory of plasticity. This disposes of the first class of rheological materials.[3] Oldroyd [1947; AMR 1, Revs. 1207, 1208] has also treated special problems of the Bingham-body.

2 The materials of the second class were first thought by Bingham and his school to exhibit also a sort of plasticity which was called "pseudoplasticity" until Reiner (1929), emphasizing their character as liquids, called them "non-Newtonian" liquids. Two different mechanisms were proposed in order to explain the variability of their viscosity defined by

$$p_{(0)rs} = 2\eta f_{(0)rs} \qquad [3]$$

One originated from Ostwald and his school (1925) who coined the term "structural viscosity." Considering that all these liquids are dispersed systems, the change of viscosity was attributed to a change of the structure of the dispersed phase (the term "structure" used in the widest sense) brought about by the change in stress or rate of flow. Reiner (1949,a) has listed in a table the different types of structural change. The problem was approached from two sides. In the "macrorheological" approach the fluidity $\varphi = 1/\eta$ was expressed as an even function of the tangential stress p_t in simple shear. The rheological equation thus postulated was integrated for different viscometers and results compared with observations. Alternatively, an empirical equation was fitted to viscometric observations and a rheological equation established accordingly by means of a method developed by Weissenberg (1929). The first method was named by Hersey (1932) "integration," the second "differentiation." These attainments are mentioned here because they are so little known that only recently, in an article, the latter method was described as a "powerful *new* method" [Clark and Deutsch, 1950].

Several rheological equations have so far been derived by these two methods, but it must be said that none has been solidly established and this remains an open problem. Generally η, being a scalar quantity, will be a function of the three invariants of the flow tensor f_{rs}, or φ of those of the stress tensor p_{rs}. The dependence of φ upon $I_p = p_{\alpha\alpha}$ has found a most thorough experimental investigation in the hands of Bridgman (1949) [cf. also Weber, 1948; AMR 1, Rev. 1579], but for obvious reasons very few liquids permit experiments by which the dependence upon the third invariant can be established, III_f vanishing in laminar flow. In connection with this question, Trouton's experiments where

[1] It is sometimes maintained that because of "wall effects" viscometric observations cannot decide between both, but Krakauer (1950) has recently provided proof to the contrary.

[2] The flow-tensor f_{rs} is defined by

$$f_{rs} = {}^1/_2(\partial v_r/\partial x_s + \partial v_s/\partial x_r) \qquad [2.1]$$

p_{rs} is the stress-tensor while the subscript (0) indicates the deviator.

EDITOR'S NOTE: The fourth in a series of articles, contributed to APPLIED MECHANICS REVIEWS by international authorities, surveying important topics in applied mechanics.

[3] The subject of plasticity will be dealt with in a separate article.

III_f does not vanish, being positive in extension and negative in compression, have again become of interest.

The second approach may be called "microrheological." Here an attempt is made to derive the rheological properties of the dispersed system from the rheological properties of its constituents on the example of Einstein's classical derivation of the "viscosity" of a dilute suspension of rigid spheres in a Newtonian liquid, a recent empirical generalization of which is by Robinson [1949; AMR **3**, Rev. 1124]. The immediate purpose of these investigations is mostly the determination of the weight of macromolecules by means of viscosity measurements of their solutions. Sadron [1949; AMR **3**, Rev. 1480] presented a review of the subject at the first International Rheological Congress. At the same Congress a contribution to the subject was read by De Bruijn [1949; AMR **3**, Rev. 1695].

3 An explanation of variable viscosity of an entirely different type was proposed by Weissenberg and school [Eisenschitz, 1931]. He considered a liquid possessing elasticity of shape similar to the elastic "solid" postulated by Maxwell, the stresses of which are subject to relaxation. Following a suggestion by Hencky (1929), Eisenschitz showed that a "Maxwell liquid" with constant viscosity would simulate a variable apparent viscosity in laminar flow due to the fact that the elements are rotated. The problem was later treated in more detail by Weissenberg (1935) and recently by Fromm [1947; AMR **2**, Rev. 202] and by Burgers [1948; AMR **2**, Rev. 739]. However, Reiner (1951,a) has shown that two different kinds of Maxwell bodies must be distinguished, namely, elastic sols where the tensor of stress is coaxial with the tensor of flow, and gels where it is coaxial with the tensor of deformation. The Weissenberg-Eisenschitz explanation of variable viscosity applies in the second case only. A complete theory on Maxwell gels is still outstanding. A recent contribution is by Fromm [1947; AMR **2**, Rev. 1030. 1948; AMR **3**, Rev. 292]. A special kind of elastic sols was investigated by Froehlich and Sack (1946), who considered elastic solid spheres dispersed in a viscous liquid, while Cerf [1947; AMR **1**, Rev. 1282] assumes the particles to be ellipsoidal. Such a system will not only possess a relaxation- but also a retardation-time. Denoting by H a (Hookean) elastic spring, by N a (Newtonian) dashpot, its "structural formula" is $Bu = N_1 - (N_2|H)$[4] (a vertical dash indicates "parallel coupling," a horizontal dash "coupling in series"), while the structural formula of the Maxwell gel is $M = N - H$. A discussion of the models corresponding to these formulas was given recently by Gubanov [1949; AMR **3**, Rev. 465].

4 The Bingham, Maxwell, Burgers body and other ideal materials were proposed in order to explain complex viscous, plastic, and elastic behavior in various combinations, but the classical viscous liquid and elastic solid were not considered as coming within the range of rheology. Recently, rheology has made inroads in both directions. Following a suggestion by Racah, Reiner (1945) has shown in the first and also [1948; AMR **2**, Rev. 574] in the second case that the most general relation between the tensor of stress on the one hand and those of flow or strain on the other hand requires the appearance of a second-order term, named cross viscosity and cross elasticity respectively, as follows:

$$p_s^{\ r} = F_0\,\delta_s^{\ r} + F_1 f_s^{\ r} + F_2 f_\alpha^{\ r} f_s^{\ \alpha} \qquad [4]$$

with the flow tensor $f_s^{\ r}$ to be replaced by the strain tensor $e_s^{\ r}$ in the case of elasticity. Cross viscosity can explain certain strange phenomena in certain liquids which were first demonstrated by Weissenberg (1947), the most striking being the climbing of the liquid up a rod which is rotated in it [cf. also Reiner, Scott Blair, and Hawley, 1949]. Following Reiner, Rivlin [1948; AMR **2**, Revs. 77, 1024. 1949; AMR **3**, Rev. 912] considered special cases of laminar flow of a liquid possessing cross viscosity, but his papers make difficult reading by his "eschewing tensorial notations and principles." This may be due to his preference for so-called "physical components," but Braun (1951) has pointed out that in any tensor equation such as [4] the mixed tensor components in orthogonal coordinates can be replaced by the physical components.

Cross elasticity arises with *large* elastic strains, the theory of which was treated by Murnaghan [1949; AMR **3**, Rev. 1048]. The measure of finite strain is not unambiguous; several measures have been proposed, which all degenerate for infinitesimal strains to an expression analogous to [2.1] when the velocity vector $\mathbf{v}_r$ is replaced by the displacement vector $\mathbf{u}_r$. Two measures form the extremes, namely, the Green-Saint Venant measure

$$_{rs}e = \tfrac{1}{2}\,[g_{\alpha\beta}(_{r},x^\alpha)(_{s},x^\beta) - {}_{rs}g] \qquad [5]$$

where indexes on the left refer to the unstrained, and on the right to the strained state, and the Almansi-Hamel measure

$$e_{rs} = \tfrac{1}{2}\,[g_{rs} - {}_{\alpha\beta}g(^\alpha x_{,r})(^\beta x_{,s})] \qquad [6]$$

It can then be shown (Reiner, 1951,b) that when either the first or the second expression for the strain tensor is introduced in the classical stress-strain relation (in which $F_2 = 0$), there results in simple shear either a tension in the direction of the displacement or a compression in the direction of the displacement gradient. In torsion of a cylinder both will result in longitudinal pressures over the cross section, which vanish at the boundary for the first and in the center for the second measure. If these pressures are not balanced by external forces the cylinder is partially lengthened. Long before rheology was thought of, Poynting (1909, 1912) had found that a steel wire lengthens on excessive torsion, which he tried to explain on the basis of the classical theory using [6]. Rivlin [1947; AMR **1**, Rev. 36], twisting a rubber cylinder between platens which prevent any change of length, found that pressures are exerted upon these platens, which Weissenberg (1949) attributed to "strangulation" effects due to stresses arising from the application of [5]. On closer examination of the experimental observations it is, however, found that both the Poynting and Weissenberg effects are present in both cases which require either the introduction of a finite coefficient of cross elasticity or the use of a measure different from both [5] and [6] and fitted to the special case. In a series of papers Rivlin [1948; AMR **2**, Revs. 841, 1491, 1492] adopted a measure derived by Guth and James (1941) from a kinetic theory of rubber elasticity. Richter [1948; AMR **3**, Rev. 838. 1949; AMR **3**, Rev. 1049] has discussed the logarithmic measure. These problems arising from "second-order" effects in viscosity and elasticity are far from settled. Truesdell has reviewed the position in two monographs (awaiting publication). He has himself generalized the Newtonian viscous liquid in a different way [1948; AMR **2**, Rev. 773]. It seems that in Weissenberg's "general liquids" (1947) both cross viscosity and cross elasticity are present, the latter in a less, the former in a more stable state [cf. Braun, Frei, and Reiner, 1950].

Very few, if any, quantitative observations have been published. Weissenberg [1949; AMR **3**, Rev. 1690] has described a comprehensive apparatus by which crucial experiments could be performed, but his paper does not include quantitative data. In the absence of such, hypotheses about the structure of the materials showing these effects lack proper foundation. It might be mentioned that Swift [1947; AMR **1**, Rev. 642] has observed in many metals length changes under torsional overstrain, a kind

[4] This ideal material may be named Burgers-body, having first been postulated by Burgers (1935).

of cross plasticity which could be interpreted as a "freezing in" of cross-elastic strains in progressive work-hardening.

5 Some problems which loomed large in the heyday of rheology have been pushed into the background. One such problem concerned "wall effects" in the flow of liquids, but recently Oldroyd [1949; AMR **3**, Rev. 1556] has again taken up this problem referring to views expressed by Reiner (1931). Another problem which was thought to have been disposed of long ago has recently come to the fore; it is the question of the magnitude of the coefficient of volume or dilatational viscosity (ζ). Stokes when deriving his equations of viscous flow assumed it to vanish, but recently statements were made to the effect that it is much larger than the shear viscosity [cf. Liebermann, 1949; AMR **3**, Rev. 1119]. It seems that generally a "liquid" may possess two kinds of volume-viscosity, one (ζ) related to volume-flow ($f_{\alpha\alpha}$), the other (ζ_κ) to volume-elasticity ($e_{\alpha\alpha}$), the rheology of the dilatation being represented by a Bu-body in which ζ corresponds to N_1 and ζ_κ to N_2. Volume-flow has been described by Reiner (1949,c; AMR **4**, Rev. 1185) and Reiner, Rigden, and Thrower (1950) in such "liquids" as concrete and asphalt. The Stokes liquid will result when $\zeta \rightarrow \infty$ and $\zeta_\kappa \rightarrow 0$, or in other words when the volume-relaxation time is infinite and the volume-retardation time vanishes [cf. also Mason, 1947; AMR **1**, Rev. 4].

6 Rheology has been treated here as applied mechanics. It is bordering on one side on the atomistic theory of matter, on the other side on psychophysics, both of which can be comprised under "metarheology." A book which belongs to the first side is by Treloar [1949; AMR **3**, Rev. 1962]. A paper dealing with the second side is by Harper [1948; AMR **3**, Rev. 1961]. This subject is extensively treated by Scott Blair (1949). Rheology finds application in both soil mechanics and lubrication. The first will be reviewed separately; for the second compare Hersey [1949; AMR **3**, Rev. 1613]. Textbooks on rheology have been published by Reiner (1949,a,b; AMR **4**, Revs. 1159, 1158) which may be amplified by Burgers and Scott Blair [1948; AMR **3**, Rev. 1939]. The connection with classical mechanics of continua can be found in Sommerfeld's books [1947; AMR **3**, Rev. 1023. 1950; AMR **3**, Rev. 1427]; the connection with structural engineering in Freudenthal's book [1950; AMR **3**, Rev. 1922].

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Weissenberg, K., cf. Rabinowitsch, B., *Z. phys. Chem. Ser. A*, **145**, p. 1, 1929.

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Communications

Concerning Rev. 1843 in this issue

The "paradox" described by the reviewer is explained following equation (4) on p. 59 and again on the top of p. 64. Local Cartesian coordinates are not used anywhere in my paper, and reviewer is certainly mistaken in suggesting that I have incorrectly used the continuity equation. The term "potential vorticity" has been current in meteorological literature for more than ten years. Mathematical notation should be clear from the context.

Paper is concerned with a phenomenon that has no direct connection with vertical motion or baroclinity, and my remarks about barocline processes were incidental and not intended to justify anything. I also fail to see the relevance of the reference to two papers dealing with the stability of three-dimensional barocline perturbations.

Reviewer speaks of a "claim." The reader may judge whether my remarks on the point in question warrant such an unequivocal interpretation. George W. Platzman

Concerning AMR 3, Rev. 1723

The reviewer's doubt regarding the continuity of the tangential acceleration across a vortex sheet was erroneous. In forming his counterexample (which was misprinted), the reviewer took no account of the continuity of pressure, from which the author's assumption can be derived. C. Truesdell

Correction to AMR 3, Rev. 1907: Campus, F., Pre-stressed concrete.

The source for this paper is *Rev. Univ. Min.* **5**, Ser. 9, 12, 421–436, Dec. 1949.

Correction to AMR 4, Rev. 335: Sears, W. R., The linear-perturbation theory for rotational flow.

The source for this paper is *J. Math. Phys.* **28**, 4, 268–271, Jan. 1950.

Correction to **AMR 3, Rev. 2414**

The last sentence should refer only to the example in part (b) of Professor Ferrari's paper and should read: Some of the methods employed therein are essentially generalizations of Sauer's method for axisymmetric flow.

The erroneous wording is due to typing omissions and the reviewer offers his apologies. G. Kuerti

Theoretical and Experimental Methods

(See also Revs. 1466, 1477, 1480, 1541, 1542, 1567, 1647, 1660, 1738, 1759, 1804, 1808, 1837)

1423. **Kharkeevich, Yu. F., The graphical solution of partial differential equations of parabolic type** (in Russian), *Prikl. Mat. Mekh.* 14, 3, 303–310, May–June 1950.

Fourier's heat equation $\partial u/\partial t = a^2\nabla^2 u$ with initial conditions and boundary conditions is reduced in the usual manner to a partial difference equation for values at the nodes of a rectangular screen. This difference equation is solved by an obvious step-by-step procedure. Three worked examples are given, one each for the one-, two- and three-dimensional problems. Actually, the work is done numerically rather than graphically.

Courtesy of Mathematical Reviews W. E. Milne, USA

1424. **Szebehely, V. G., An analogy between open soap bubble and compressible flow,** *J. aero. Sci.* **17**, 12, p. 809, Dec. 1950.

1425. **Donsker, M. D., and Kac, M., A sampling method for determining the lowest eigenvalue and the principal eigenfunction of Schrödinger's equation,** *J. Res. nat. Bur. Stands.* **44**, 5, 551–557, May 1950.

This is a preliminary report on a sampling method, constituting a sophisticated application of Monte Carlo methods, for finding the lowest eigenvalue and corresponding eigenfunction of Schrödinger's equation. The method may be compared and contrasted with that of Metropolis and Ulam ["The Monte Carlo method," *J. Amer. statist. Assn.* **44**, **247**, pp. 335–341, Sept. 1949], in that here also a random walk process alone is used, without the further multiplicative process used in paper referred to. The one-dimensional Schrödinger eigenvalue problem is studied after several decisive simplifications. The extension to higher dimensions by these methods is immediate. The Monte Carlo aspect consists in the calculation of two selected one-dimensional distributions from the limiting two-dimensional-distribution function used in the Schrödinger problem. To date the study has been carried out only for nonnegative potential functions. A simple formula is obtained. Numerical examples with tabular results are discussed.

Albert A. Bennett, USA

1426. **Hamaker, H. C., Error theory and mathematical statistics; the rounding-off of observations** (in Dutch), *Ingenieur 's-Grav.* 61: 24, 25; A 227–231, A 235–241, June 1949.

The problem investigated is how far calculation of the standard deviation can be simplified by using rounded-off values. By considerations of mathematical statistics valid not only for the normal Gaussian-error law but also for general error distributions, author obtains the following rules of application for practical use: The order of magnitude of round-off error is one half of the standard deviation (for very few material observations); $^1/_6$ of the variation width determined by 5 or 10 observations; $^1/_6$ of the maximum difference formed by pairs from a series of 10 pairs of observations. Simplification occurs only when the round-off range is at least $^1/_5$ of the maximum amount which is obtained according to the prescription above. The round-off range for the averaged n observations values has to have the order of magnitude $1/(n)^{1/2}$ × round-off range of the single observation value.

Manfred Schäfer, Germany

1427. **Thomson, W. T., Matrix solution for the vibration of nonuniform beams,** *J. appl. Mech.* **17**, 3, 337–339, Sept. 1950.

Paper is based on a method of analysis published by the reviewer in 1944, only difference being that the principal equations of the earlier paper are presented in matrix form and the tabular calculation is replaced by a matrix equation. At first the new method seems to be superior because of the condensed form of presentation made possible by matrix method. However, anyone working out a numerical example by each of the two methods will discover that new method requires much more time than does the old one. N. O. Myklestad, USA

1428. **Dubnov, Ya. S., Foundations of vector calculus. Part 1. Vector algebra. Elements of vector analysis. 4th ed. [Osnoví vektornogo ischisleniya. Chast I. Vektornaya algebra. Elementí vektornogo analiza],** Gosud. Izdat. Tekhn.-Teor. Lit., Moscow-Leningrad, 1950, 368 pp.

Chapter headings are: Affine correspondences between vectors; Scalar multiplication; Vector multiplication; Vector functions of a scalar variable, scalar fields; Differential properties of curves.

1429. **Einsporn, Critique of measuring methods** (in German), *Z. angew. Math. Mech.* **29**, 9, 282–283, Sept. 1949.

1430. **Blackshear, Perry L., Jr., Sonic-flow-orifice temperature probe for high-gas-temperature measurements,** *Nat. adv. Comm. Aero. tech. Note* 2167, 20 pp., Sept. 1950.

Author describes and analyzes the subject apparatus and compares its performance with the standard shielded thermocouple and the sodium D-line reversal method. Calibration of the orifices is described and results of measurements in air and in propane-air combustion products up to temperatures of 4000 R are discussed. Schematic diagrams are presented together with curves illustrating performance of the apparatus. References to similar works are given. E. Cartotto, USA

1431. **Bubb, F. W., Nisle, R. G., and Carpenter, P. G., An electronic analog computer for solving the flash vaporization equilibrium equation,** *J. Petr. Technol.* **21**, 5, 143–148, May 1950.

The computer is designed to solve the pair of equations

$$\sum_{m=1}^{n} \frac{z_m}{[1+(K_m-1)\nu]} = \sum_{m=1}^{n} \frac{K_m z_m}{[1+(K_m-1)\nu]} = 1$$

for ν, the mol fraction of vapor in a vapor-liquid multicomponent system in equilibrium, and to compute the terms of these sums. Here K_m is an equilibrium constant for the m^{th} component, and z_m is the mol fraction of the m^{th} component in the mixture, all of these being assumed known for each computation. The seven basic computer units are resistance networks with voltage matching servomechanisms. Operation of these basic units is described, as well as their integration into the complete computer. Several applications are discussed: Determination of optimum separator operating conditions, evaluation of reserves, and natural-gasoline plant calculations. Results of error studies and a method of seeking a refined answer are included. In common with other computers of this type, there are many detailed operating considerations which must be observed for optimum performance. Paper is not intended as a manual of operation, however, and

these details are properly passed over. Although the point is not made explicitly, this computer is a good illustration of the practicability of treating with special-purpose computers the routine but time-consuming computations which arise in industrial processes. Robert E. Roberson, USA

1432. Woods, L. C., Improvements to the accuracy of arithmetical solutions to certain two-dimensional field problems, *Quart. J. Mech. appl. Math.* **3,** part 3, 349–363, Sept. 1950.

Paper describes two methods of improving the accuracy of arithmetical solutions on a square mesh of two-dimensional partial differential equations. Alternative to reducing size of the mesh, same accuracy is obtained with less labor on a coarse mesh by (a) using a higher-order difference equation as a closer approximation of the differential equation, or by (b) calculating a correction term from the higher differences to apply to results obtained by using the first approximation. Methods are illustrated by an example of Poisson's equation with comparison of accuracies. W. T. Thomson, USA

1433. Rumberg, Alfred, Nomograms for reduction of gas humidity measurements (in German), *Arch. tech. Messen* no. 169, p. T13, Feb. 1950.

1434. Van Wijngaarden, A., Table of the cumulative symmetric binomial distribution, *Proc. K. Ned. Akad. Wet.*, 12 pp., 1950.

Table of the function $P(n,c) = 1 - 2^{1-n}\sum_{s=0}^{c}\binom{n}{s}$. Ed.

1435. Gross, B., Note on the inversion of the Laplace transform, *Phil. Mag.* **41** (7), no. 317, 543–544, June 1950.

Author presents Titchmarsh's inversion of Stieltjes form of iterated transform, which enables one to extend the scope of existing tables of Laplace transforms for a certain class of functions. Results are summarized as follows: If $f(s) = \mathfrak{L}F(t)$, then $\mathfrak{L}^{-1}F(s) = (1/\pi)\text{Im}\, f(te^{-i\pi})$. An example is included to illustrate the procedure. W. T. Thomson, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 1455, 1463, 1551, 1861)

1436. Schorr, W. E., Cable-pulley friction, *Trans. Amer. Soc. mech. Engrs.* **72,** 8, 1173–1180, Nov. 1950.

Factors contributing to friction in cable-pulley or rope systems are analyzed, and an empirical equation for calculating this friction is derived. From author's summary

1437. Mazet, R., On the landing impact phenomenon (in French), *Proc. seventh int. Congr. appl. Mech.* **4,** 109–120, 1948.

Starting from results obtained in his previous papers, author investigates the general conditions for impact to be possible in a mechanical system having connections which contain restraints. He establishes these conditions for plane motion (especially in case when there is rolling without sliding) and then applies them to an airplane rolling on the ground, resting partly on air, partly on its main undercarriage except for the nose wheel, and maintained parallel to the ground by the pilot manipulating the elevator. Author arrives at the result that impact is not to be expected in modern airplanes, even without shock absorbers. This is not true for airplanes of flying-wing type if the runway is very rough. Under these conditions, it could happen that in the last moments of take-off the rolling becomes unstable and transforms into impact. Z. Horák, Czechoslovakia

1438. Diamond, E. L., and Frankau, A. M., The effect of tapered treads on the motion of overhead travelling cranes, *Instn. mech. Engrs. Proc.* **162, 3,** 313–320, 1950.

Tapered treads are advantageous for reducing wear and tear on cranes and rails. The known theory of motion of a four-wheeled vehicle having pairs of tapered-tread wheels joined by an axle is applied to cranes, and is extended to the important case of a four-wheeled crane with only one axle and to eight-wheeled cranes with two pairs of wheels driven by a common shaft. Observations of actual motion of a number of cranes with parallel and tapered-tread wheels are recorded. Tapered-tread cranes give sinusoidal curves of motion in good agreement with theory, and design rules for width of both parallel and tapered treads are suggested.

From authors' summary by Peter Kyropoulos, USA

1439. Obmorshev, A. N., Investigation of phase trajectories at infinity (in Russian), *Prikl. Mat. Mekh.* **14,** 383–390, 1950.

The Poincaré system (1) $dx/dt = P(x,y)$, $dy/dt = Q(x,y)$ is investigated for a limit cycle at infinity in the following manner: The phase plane is projected onto a hemisphere H of radius one tangent to the plane at the origin, then M is projected orthogonally on the plane. If ρ, θ are the polar coordinates this yields the transformation

$$x = \rho\cos\theta/(1-\rho^2)^{1/2};\quad y = \rho\sin\theta/(1-\rho^2)^{1/2}$$

and it leads from (1) to (2)

$$d\rho/d\theta = \rho(1+\rho)(1-\rho)^m(\Phi(\rho,\theta)/\Psi(\rho,\theta))$$

with Φ, Ψ analytic and not divisible by $1-\rho$.

Infinity is now imaged into C, the circle of radius one. If $\Psi \equiv 0$ on C and $m>0$, C is a limit cycle. If $m = 0$, C is not a trajectory. Finally, if $m<0$, C is a closed curve without contact.

Stability and the existence of limit cycles $\to C$ are discussed. Application is made to the study of the trajectories of the system (1) corresponding to the following equation derived from the coupling of a series generator with an independently excited motor:

$$\ddot{x} - \mu[1/(1+x^2) - \nu]\,\dot{x} + x = 0$$

References, besides the classical writings of Poincaré, Bendixson, and Liapounov: Petrovski, *Mat. Sborn.* **41,** 1934; von Mises, *Comp. Mat.* **6,** 1938; Lefschetz, "Lectures on differential equations," *Ann. Math. St.*, no. 14, p. 142 (reference not known by author of paper). S. Lefschetz, USA

1440. Maĭzel', A. D., On stability in the first approximation (in Russian), *Prikl. Mat. Mekh.* **14,** 171–182, 1950.

Let $P(t)$ be an $n \times n$ matrix whose elements $p_{sk}(t)$ are continuous and bounded for $t \geqq t_0$; let $L(\mathbf{x}, t)$ be a vector function of the vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, analytic in the x_i, vanishing together with its first partial derivatives at the origin for all $t \geqq t_0$. The system of differential equations (1) $d\mathbf{x}/dt = P(t)\mathbf{x} + L(\mathbf{x}, t)$ has m-fold stability at the origin if there is a manifold M of dimension $\geqq m$ such that $\lim_{t\to\infty}\mathbf{x}(t) = 0$ for all solutions $\mathbf{x}(t)$ having their initial values $\mathbf{x}(t_0)$ in M.

The author obtains a complicated but powerful sufficient condition for m-fold stability. Let the linear system $d\mathbf{x}/dt = P(t)\mathbf{x}$ possess m independent solutions whose characteristic numbers $\lambda_1, \ldots, \lambda_m$ are positive [cf. Liapounoff, A., "Problème générale de la stabilité du mouvement," Princeton, N. J., Princeton University Press, 1947, especially p. 224]. Suppose that positive $k < \min(\lambda_1, \ldots, \lambda_m)$, and a constant N exist such that for any continuous vector $\omega(t)$ whose components satisfy $|\omega_i(t)| < e^{-kt}$ the system $d\mathbf{y}/dt = (P(t) + kI)\mathbf{y} + \omega(t)$ has a solution $\mathbf{y}(t)$ whose components are bounded by N for $t \geqq t_0$. Then (1) has m-fold

stability at the origin. Following the formulation and proof of this theorem, it is shown that author's stability criterion includes results of Liapounoff [op. cit., p. 254] and Malkin.

Courtesy of Mathematical Reviews J. G. Wendel, USA

Gyroscopics, Governors, Servos

1441. **Jones, Arthur L., and Briggs, Benjamin R., A survey of stability analysis techniques for automatically controlled aircraft,** *Nat. adv. Comm. Aero. tech. Note* 2275, 112 pp., Jan. 1951.

A survey of the stability-analysis techniques for automatically controlled aircraft is presented. The survey is limited to the techniques commonly applied to linear, continuous-control systems wherein the difference between the output and input quantities is measured continuously and is used in the operation of the system (a closed-loop system). An evaluation of the techniques, based on the kind and amount of information derivable, is included. An illustrative example is also presented to demonstrate the calculations involved for a typical aircraft-autopilot combination.

A. Vazsonyi, USA

1442. **Heye, B. F., Power plant controls,** *Instruments* **24**, 1, 38–41, 55–57, 82–83, Jan. 1951.

Interestingly written but elementary discussion of the major controls necessary for operation of present-day steam power plants. Operation of the various controls is illustrated by a number of diagrams, but details and method of operation are covered superficially. Some of the terminology seems slightly garbled but general description is good. Little fundamental information is given, but article should be interesting and useful to those concerned with the over-all picture of power-plant control.

A. O. White, USA

1443. **Johnson, R. L., and Rea, J. B., Importance of extending Nyquist servomechanism analysis to include transient response,** *J. aero. Sci.* **18**, 1, 43–49, Jan. 1951.

The serious error possible in estimating stability and response of high-order linear controls by use of rule of thumb in inspection of the Nyquist diagram is illustrated by an example of a guided-missile control. The approximation of the actual control by a second-order system having the same maximum amplitude and phase contours (M curves) was shown to be very bad in the example cited.

A method of computing response in the time domain, using the frequency-response characteristics, is described. A frequency spectrum of the input is computed and multiplied by the frequency-response characteristic of the control. Fourier synthesis by summation of the output frequency components then yields output of the system in the time domain.

Reviewer comments that a striking illustration of the error possible when using M curves in designing high-order systems is illustrated. Digital computation of time response from frequency response appears to be the most foolproof method. Analog computers, however, are also useful and quicker. A high-order differential equation is chosen that best represents the frequency response of the control. Constants for this equation are then used in the analog.

John C. Sanders, USA

1444. **Walters, E. R., and Rea, J. B., Determination of frequency characteristics from response to arbitrary input,** *J. aero. Sci.* **17**, 7, 446–452, July 1950.

A method is given for determining the frequency-response spectrum of a simple dynamic system when an arbitrary driving force (input) and the corresponding response (output) are known. The complex frequency response (transfer function) is determined as the ratio of corresponding terms in the Fourier expansions of the driving force and response. The numerical evaluation of the Fourier coefficient integrals is shown to be equivalent to a simple matrix multiplication. Actual computation may be done rapidly on automatic calculating equipment, and an example has been carried out.

A. Vazsonyi, USA

1445. **Heinrich, G., On the theory of the course gyroscope** (in German), Wien, Franz Deuticke, *Federhofer-Girkmann-Festschrift*, 205–218, 1950.

A frictionless cardanic supported body rotating along its axis with high angular velocity maintains the direction of its axis relative to a cosmic inertial system, performing thus a precessional motion with respect to the rotating earth. A course gyroscope is obtained when counteracting externally applied force moments are introduced, eliminating (compensating) this relative motion.

Basing his derivations on work done by various other investigators, author presents a complete and strictly mathematical theory of the course gyroscope, deducing its differential equations of motion and closed expressions for the force moments mentioned above. His method of presentation is brilliant and extremely clear.

Max A. Dengler, USA

1446. **Gaden, D., On the automatic governing of hydraulic turbines: influence of certain intervening characteristics on stability conditions** (in French), *Houille blanche*, no. spec. B, 717–723, 1949.

By means of Nyquist's method, author shows in vectorial diagrams the unstabilizing effect of water hammer on governing, which is inherent in water turbines even if they are equipped with the so-called double governing system. With the same method, he then demonstrates that the stability is improved if the driving torque falls when the speed rises, or if the opposing torque rises under the same circumstance. To back this latter favorable influence, author proposes that the generator voltage regulator be designed in such a manner that it maintains constant voltage only if frequency is constant, but raises it in acceptable limits if frequency rises, and vice versa.

Miroslav Nechleba, Czechoslovakia

1447. **Bower, John L., A note on the error coefficients of a servomechanism,** *J. appl. Phys.* **21**, 7, p. 723, July, 1950.

This short note presents a method for expanding the error function of a linear servomechanism into the input function and its higher derivatives.

A. Vazsonyi, USA

1448. **White, Roland J., Investigation of lateral dynamic stability in the XB-47 airplane,** *J. aero. Sci.* **17**, 3, 133–148, Mar. 1950.

Standard studies of lateral stability indicated that the increase in wing loading and in altitude of operation of modern airplanes leads to poor damping of the oscillatory yawing-rolling mode (Dutch roll) especially for sweptback airplanes such as XB-47. Pilots confirmed the predictions, objecting particularly to the rolling motion associated with the high dihedral effect. Variation of parameters showed that a most effective improvement can be obtained by increasing the damping yawing moment due to yawing, C_{n_r}. If this is done by changing tail geometry, compensating changes in other parameters essentially defeat the cure. Paper reports on the successful analytical and physical development of a yaw rate gyro operating the rudder (in conjunction with pilot and hydraulic boost), thereby increasing the parameter C_{n_r} alone.

Analysis adopts the servomechanistic approach of inverse frequency-response characteristics. However, in addition to establishing conditions for stability of the combined system, an

actual estimate of the degree of damping is made. This is done by reducing the degrees of freedom of the problem from three to one by a series of plausible and partially verifiable assumptions (for the case of Dutch roll).

High-speed hunting instability developed when yaw damper was installed. Two separate causes were isolated and satisfactorily analyzed: Nonlinear characteristics of the servomechanism at small amplitudes, and fuselage side-bending (distorting the gyro feel).

Careful "yaw-damper" design thus improved remarkably both the Dutch roll and spiral stability of the XB-47 airplane. Work on the one negative feature reported, namely, increased pedal forces in steady turns, is continuing. Report is stimulating and clearly presented; a reader without background in servomechanisms will not be lost. M. V. Morkovin, USA

1449. Solodovnikov, V. V., On the application of trapezoidal frequency characteristics to the analysis of the behavior of systems of automatic regulation (in Russian), *Avtomatika i Telemekhanika* **10**, 362–376, 1949.

Author considers a differential equation of an automatic regulator and assumes that solution of this equation by Laplace transform method leads to the expression

$$x(t) = 2\pi^{-1} {}_0\!\int^{\infty} \omega^{-1} R(\omega) \sin t\omega d\omega \qquad [1]$$

where $x(t)$ is the unknown function and $R(\omega)$ (generalized real frequency characteristic) is a known function vanishing for sufficiently large values of ω. He proposes a method for a rapid determination of the properties of $x(t)$. This method consists of approximating $R(\omega)$ by a linear combination of "trapezoidal functions" $\tau(\omega)$ which are positive and constant for $0 < \omega < \omega_1$, linear for $\omega_1 < \omega < \omega_2$, and vanish for $\omega > \omega_2$. If $R(\omega) = \tau(\omega)$ the integral [1] can be expressed in closed form. For $\tau(0) = 1$, $\omega_t = 1$,

$$x(t) = 2\pi^{-1} \{\mathrm{Si}(\kappa t) + (1 - \kappa)^{-1} [\mathrm{Si}(t) - \mathrm{Si}(\kappa t)t^{-1} (\cos t - \cos \kappa t)]\} \qquad [2]$$

where $\kappa = \omega_1/\omega_2$. Using this formula, $x(t)$ may be found for any trapezoidal function $\tau(\omega)$. In particular, it is proposed to approximate $R(\omega)$ by a "typical function" which is a difference of two trapezoidal functions. Paper contains tables and graphs of the functions [2] and two examples. L. Bers, USA

Vibrations, Balancing

(See also Revs. 1427, 1439, 1545, 1546, 1550, 1861)

1450. Ayre, R. S., and Jacobsen, L. S., Natural frequencies of continuous beams of uniform span length, *J. appl. Mech.* **17**, 4, 391–395, Dec. 1950.

Authors present graphical network for rapid determination of roots of frequency equation as function of number of spans and when ends of beam are (a) simply supported, (b) clamped, (c) one clamped and one simply supported. Factor is given to account for shear distortion and rotatory inertia. H. D. Conway, USA

1451. Buckens, F., Influence of the relative radial thickness of a ring on its natural frequencies, *J. acous. Soc. Amer.* **22**, 4, 437–443, July 1950.

Flexural vibrations of a ring in its plane are studied by taking shear effect into account, and it is proposed to compute the deviation from the classical frequency formula which is only valid for very small ratios of thickness to diameter. In a second chapter, the possibility of extension is included, and, in this procedure, the frequencies of extensional vibrations are obtained as a by-product. Shear effect is shown to play a predominant role in the deviation from the classical formula and is more important than curvature effects and the rotational inertia. Extensibility influence is found to be very small. An interesting comparison can be made with the similar influence of secondary factors on the flexural vibration frequencies of a straight beam.

From author's summary by Sven T. A. Ödman, Sweden

1452. Waller, Mary D., Vibrations of free elliptical plates, *Proc. phys. Soc. Lond. Sec. B*, **63**, part 6, 366B, 451–455, June 1950.

Systematic observations were made on normal vibrations of free elliptical plates, using two plates with the ratio of axes 2/1 and 5/4, respectively. Relative frequencies of normal vibrating modes and photographs of the nodal lines are assembled in tables. The observations confirm Chladni's results. The nodal systems are divided into the same four classes of symmetry as those of rectangular plates and the correspondence between vibrations of elliptical, rectangular, and circular plates is traced.

From author's summary by Sven T. A. Ödman, Sweden

1453. Sauer, F. M., and Garland, C. F., Performance of the viscously damped vibration absorber applied to systems having frequency-squared excitations, *J. appl. Mech.* **16**, 2, 109–117, June 1949.

1454. Clarion, Claire, On the damping of small oscillations of a heavy viscous liquid in a U-tube (in French), *C. R. Acad. Sci. Paris* **230**, 22, 1926–1928, May 1950.

In order to check the theory of small oscillations of viscous liquids in U-tubes, tubes with a diameter $2R = 2.1$ cm and 1.6 cm and liquids with a kinematic viscosity ν between 0.00118 and 0.095 cm²/s are used. Let $\omega = \Omega + i\lambda$ be the complex period, $\omega_p = (2g/L)^{1/2}$, L = length of the liquid column, then the theory demands that λ/ω_p and Ω/ω_p be dependent only on $R^2\omega_p/\nu$. The measurements of ν/ω_p are in good agreement with theory in the investigated range of $R^2 \cdot \omega_p/\nu$ between 5–500; the values of Ω/ω_p, which were not measured very precisely, are, however, systematically below the theoretical values.

M. Herbeck, Germany

1455. Serbin, H., Periodic motions of a non-linear dynamic system, *Quart. appl. Math.* **8**, 3, 296–303, Oct. 1950.

Author gives another proof for the existence of a unique periodic solution of the generalized Lienard equation: $\ddot{x} + f(x)\dot{x} + g(x) = 0$ and a method to calculate upper and lower bounds for the amplitude of that solution. Assumptions are slightly different from those of Lefschetz, but no symmetry for $f(x)$ and $g(x)$ is required. Dario Graffi, Italy

1456. Krall, Giulio, Forced or self-excited vibrations of wires, *Proc. seventh int. Congr. appl. Mech.* **4**, 221–225, 1948.

Paper presents a theoretical study of small oscillations of electrical transmission-line conductors. Analysis is simplified by considering the line conductor as a straight tightly stretched string. Forced and damped oscillations are considered.

Louis A. Pipes, USA

1457. Zheleztsov, N. A., On the theory of the symmetric multivibrator (in Russian), *Zh. tekh. Fiz.* **20**, 788–797, 1950.

The symmetrical multivibrator of Abraham-Bloch has been discussed in sketchy manner in the "Theory of oscillations" of

Andronow-Chaĭkin [English ed., Princeton Press, 1949, p. 274]. The basic equations may be reduced, with suitable choice of the variables x, y and time unit t, to the form of two symmetrical equations in x and y of which one is

$$\dot{x} = \frac{x - k\,\varphi'(y)y}{k^2\varphi'(x)\,\varphi'(y) - 1}$$

The function $\varphi(x)$ is the characteristic of the vacuum tubes. Author assumes for it a piecewise linear approximation such that $\varphi'(x) = 0$ for $|x| > 1$ and $\varphi'(x) = 1$ for $|x| < 1$. He studies as loc. cit. the discontinuous oscillations, which arise only when $k < 1$. As a consequence in the Andronow-Chaĭkin diagram, fig. 248, the two ovals become polygons, the interior one being a square with sides parallel to the axes. Under these simplifying assumptions it is shown that there is a unique discontinuous oscillation and that it is stable [see Andronow-Witt, *Dokladĭ Akad. Nauk SSSR*, p. 189, 1930]. S. Lefschetz, USA

1458. **Klotter, K., The amplitude-time diagram of a simple beat** (in German), *Z. angew. Math. Mech.* **30**, 5/6, p. 190, May-June 1950.

When two sine waves of equal amplitude produce beats, the resulting oscillation has varying amplitude and changes phase every time the amplitude becomes zero. This change of phase is not accompanied by a discontinuity of tangent as is shown for instance in the figure illustrating beats in K. W. Wagner's book "Vibrations and wave" [Wiesbaden, 1947]. Reviewer regrets that figs. 1 (faulty) and 3 (correct) do not show clearly the essential difference and that author has not indicated clearly the parabolic character the curve has in the point discussed.

R. Vermeulen, Holland

1459. **Mendelson, Alexander, and Gendler, Selwyn, Analytical determination of coupled bending-torsion vibrations of cantilever beams by means of station functions,** *Nat. adv. Comm. Aero. tech. Note* 2185, 62 pp., Sept. 1950.

Calculation procedure, based on Rauscher's station function method [AMR **3**, Rev. 1762], is presented for finding uncoupled and coupled bending torsion vibration modes and frequencies of cantilever beams. Beam is divided into n equal parts, properties are assumed constant within each interval, and a determinantal equation (of order n for uncoupled, $2n$ for coupled vibrations) is written with the help of numerical data (tabulated up to $n = 8$). For a uniform beam, examples show that $n - 1$ frequencies are accurately obtained with an nth order determinant; authors state, however, that comparable accuracy for nonuniform beams may require higher-order determinants.

Bernard Budiansky, USA

1460. **Lewis, Robert C., and Wrisley, Donald L., A system for the excitation of pure natural modes of complex structure,** *J. aero. Sci.* **17**, 11, 705–722, 735, Nov. 1950.

Theory, construction, and operation of a multiple shaker system of exciting pure natural modes of complex structures are presented. Basic idea, which is checked on a simple test structure with variable damping provided electromagnetically, is to apply exciting forces proportional to the inertial loading and in phase with the velocity at many points on the structure. Full scale equipment involving 24 controllable shakers was designed, constructed, and applied to the ground testing of an actual airplane. Initially, the structure is excited with only one shaker until a natural frequency is roughly found in the usual fashion. Additional shakers are introduced individually with their force output made proportional to the inertial loading at their point of application by means of an accelerometer coupled directly to the shaker. The equipment can be controlled from a single console at which the phase, frequency, and amplitude at each shaker can be determined. Paul A. Libby, USA

1461. **Ashley, Holt, and Haviland, George, Bending vibrations of a pipe line containing flowing fluid,** *J. appl. Mech.* **17**, 3, 229–232, Sept. 1950.

Analytical investigation of free vibrations and forced motions caused by cross winds acting on large above-ground oil pipe lines. Relative importance of self-induced, transient, and forced vibrations could be clarified. Results important only in the negative sense of explaining why important oscillations (similar to "galloping transmission lines" or von Kármán vortex motions) did not materialize. Idea of damping with fluid flow is novel and could be of general engineering importance. Reviewer suggests a closer analysis in setup amenable to simple analytical treatment and capable of being checked experimentally.

E. G. Fischer, USA

1462. **Wintner, Aurel, On free vibrations with amplitudinal limits,** *Quart. appl. Math.* **8**, 1, 102–104, Apr. 1950.

In matrix notation let [1] $\dot{x} = A(t)x$ be a set of linear differential equations with variable coefficients, and let $r(t) = |x(t)|$ denote the "amplitude" of the solution. Then the following statement is proved: The amplitude $r(t)$ of every nontrivial solution vector $x = x(t)$ of [1] tends to a finite, nonvanishing limit, as $t \to \infty$, provided

$$\lim_{T\to\infty} \int \lambda(t)dt \quad \text{and} \quad \lim_{T\to\infty} \int^T \mu(t)dt$$

are convergent; here λ and μ denote the least and the greatest eigenvalue of the symmetrical matrix $(A + A')/2$, with A' being the transposed matrix to A. K. Klotter, Germany

1463. **Boggis, A. G., An accelerometer for measuring ship hull vibrations,** *J. sci. Instrum.* **27**, 8, 212–214, Aug. 1950; also *Shipbuilder* **58**, 507, 27–28, Jan. 1951.

Paper describes an accelerometer which has been developed for measuring the vibrations of ships' hulls. High sensitivity is obtained by use of wire-resistance strain gages of high resistance. When an accelerometer with d-c excitation is used in conjunction with an a-c amplifier, a balancing system is not required; furthermore, the accelerometer may be used in any plane without adjustment. Recording is by means of a pen recorder.

From author's summary

1464. **Nardini, Renato, On quasi-harmonic vibrations of a dissipative system with periodic elasticity** (in Italian), *Boll. Un. mat. Ital.* (3), **4**, 370–373, 1949.

Same problem as in paper reviewed in AMR **4**, Rev. 85. The condition $c < m$ is removed and a similar but more complicated inequality is obtained showing that if T is sufficiently large, then stability takes place.

Courtesy of Mathematical Reviews F. Bohnenblust, USA

1465. **Schwesinger, G., On one-term approximations of forced nonharmonic vibrations,** *J. appl. Mech.* **17**, 2, 202–208, June 1950.

In harmonically excited mechanical systems, nonlinearities often occur in damping or in the springs, while in electrical engineering the most common case is that of a nonlinear inductance (mass). For such systems with one degree of freedom a general approximating method is developed. It is assumed that the mass provides a one-term harmonical motion $x = A \sin \omega t$. As this

assumption does not fulfill the differential equation, there must be added to the exciting force $P = P_1 \sin \omega t + P_2 \cos \omega t$ a disturbing force Q in order to satisfy the equation. The closest approximation of the actual vibration will be obtained when the integral of the square value of the disturbing force Q during one cycle is a minimum. This minimum condition can be satisfied by proper choice of the unknown amplitude and unknown phase difference between the exciting force P and displacement x or, assuming the amplitude and phase difference as known but the exciting force as unknown, by proper choice of P_1 and P_2. By this latter original artifice it is comparatively easy to solve approximately every symmetrical system. The solution of asymmetrical systems can also be achieved in this way, but the procedure is considerably less convenient and the result less precise. Method, furthermore, gives approximately the error of the suggested one-term solution and thus enables dependable limits to be established for its application. Comparison with other one-term approximations and also with exact analytical solutions shows the superiority of the new method. Pavel Kohn, Czechoslovakia

1466. Stepanov, V. V., On the solutions of a linear equation with periodic coefficients in the presence of a periodic disturbing force (in Russian), *Prikl. Mat. Mekh.* **14,** 3, 311–312, May-June 1950.

Some results concerning resonance are obtained for the equation $\ddot{x} + p(t)x = f(t)$, p and f periodic, by means of Fourier series techniques. A typical theorem is: Let the homogeneous equation have bounded solutions, and let $f(t) = ae^{i\lambda t}$, $p(t) = p(t + 2\pi)$. Writing $x_1 = e^{i\alpha t}\varphi_1(t)$, $x_2 = e^{-i\alpha t}\varphi_2(t)$, φ_1 and φ_2 periodic with period 2π, as a fundamental pair of solutions of the homogeneous equation, where α is not an integer, then resonance can occur only if $\lambda + \alpha$ or $\lambda - \alpha$ is an integer.
Courtesy of Mathematical Reviews J. G. Wendel, USA

Wave Motion, Impact

(See also Revs. 1484, 1854)

1467. Pastori, M., Propagation of waves in anisotropic media and the respective principal directions (in Italian), *Nuovo Cim.* (9) **6,** 187–193, 1949.

This paper concerns the propagation of waves (surfaces across which the velocity is continuous but the velocity gradient suffers a discontinuity) in anisotropic elastic media, where the stress p_{rs} and the infinitesimal strain ξ^{ik} are related by the generalized Hooke's law $p_{rs} = c_{rsik}\xi^{ik}$. Let n^j be the wave normal, let ρ be the density, and let the propagation tensor A_{ik} be given by $-\rho A_{ik} \equiv C_{ijkh}n^j n^h$. The classical result of Hadamard ["Leçons sur la propagation des ondes," Hermann, Paris, 1903, pp. 245, 252] states that for a wave with given normal n^j, the discontinuity vector must be parallel to one of the three (or possibly infinitely many) principal directions of A_{ik}, and that to each of these there corresponds a definite real speed of propagation. Author proposes that instead of considering the wave normal as given, one may simply investigate the intrinsic properties of the elastic medium. To this end he introduces the symmetric tensor B_{rs} defined by $-\rho B_{rs} \equiv C^i{}_{ris}$, whose principal directions he calls the principal directions of the medium at the point in question, and proves that for these directions the sum of the squares of the three speeds of propagation has a stationary value relative to other directions. When the medium is of Green's type, in which the elastic tensor reduces to the form $C_{irks} = - Ca_{ir}a_{ks} + c^{jh}(\epsilon_{jis}\epsilon_{hrk} + \epsilon_{jrs}\epsilon_{hik})$, where C is a scalar, a_{ij} is the metric tensor, and c^{jh} is a symmetrical tensor, then the principal directions of the medium coincide with the principal directions of c^{jh}, which are the principal directions of Fresnel's ellipsoid. The paper closes with the remark that all invariants of A_{ij} are symmetric functions of the three speeds of propagation for the direction in question, while all invariants of B_{rs} are symmetric functions of the three sums of squares of the three speeds of propagation for the three principal directions. C. Truesdell, USA

1468. Tartakovskiĭ, B. D., On the theory of the propagation of plane waves in homogeneous layers (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **71,** 465–468, 1950.

Calculation of the coefficients of reflection and refraction of plane waves in passage through n homogeneous plane layers reduces to the solution of a system of algebraic equations of order $2(n + 1)$. These equations are obtained by putting the expressions for the wave potentials in the boundary conditions. The various resistances to passage of the waves are specified, as well as layer thickness. Author investigates the case of acoustic waves in liquid layers, characterized by their densities, velocities of propagation of acoustic waves, and thicknesses. He constructs the determinants of the system and seeks the simplest ways for the calculations. For two particular cases, calculations are made completely.
Courtesy of Mathematical Reviews V. A. Kostitzin, France

1469. Thompson, Philip Duncan, The propagation of small surface disturbances through rotational flow, *Ann. N. Y. Acad. Sci.* **51,** 463–474, 1949.

The general theory for two-dimensional gravity waves of small amplitude in an already established current of varying velocity in the depth is derived. The frequency equation relating the phase speed, wave length, and depth of water is studied for the case of linear variation in current speed with depth (which means a constant shear force in an originally laminar flow). It is recommended that other types of current velocity distribution in the depth coordinate be approximated by piecewise linear distributions with subsequent fitting of the solutions across the discontinuities. Theory is next specialized for the case of long waves in shallow water. Estimates for phase speeds are obtained, which in turn would be useful in carrying out calculations in more general cases.
Courtesy of Mathematical Reviews J. J. Stoker, USA

1470. Brekhovskikh, L. M., Refraction of plane waves from layer-heterogeneous media (in Russian), *Zh. tekh. Fiz.* 19, 10, 1126–1135, Oct. 1949.

Special feature of the theory is that it is based not on the wave equation but on the equation (of first order) for the refraction coefficient. Two methods of successive approximation are used permitting representation of the refraction coefficient in form of convergent series. An example illustrates how quickly the series converge. Translation from author's summary

1471. Thornton, D. Laugharne, Applications of stress propagation in civil engineering, *Engineering* **169,** 4403, 689–692, June 1950.

Paper summarizes the results of the theory of stress propagation based on general wave equation and their bearing on engineering problems. Main items are: (1) Propagation of pressure pulses in bars starting either from one end or from a structural connection at an intermediate point. Only extensional deformations are produced, their propagation velocity being a constant for a given material. (2) Propagation of extensional and distentional pulses in beams. Flexural deformations of cantilevers are considered in detail. Dispersion appears in consequence of distentional component of deformation, short waves traveling faster

than long ones. Internal damping affects differently waves of different frequencies, and therefore produces distortion. (3) Spherical extensional and distentional waves in three-dimensional solids and liquids. Conditions studied approximate those associated (a) with detonation of an explosive in solid rock, and (b) with under-water explosions. Numerical applications are considered in all instances.

B. Gross, Brazil

1472. Morse, R. W., The velocity of compressional waves in rods of rectangular cross section, *J. acoust. Soc. Amer.* **22, 2,** 219–223, March **1950.**

An exact solution for the velocity of elastic waves along rods is derived from the exact equations of the theory of elasticity for case of an infinitely long rectangular rod whose width $2d$ is greater than the thickness $2a$. It is assumed that the rod is homogeneous and isotropic. By focusing attention only on the boundary requirements at surfaces $x = \pm a$, holding in abeyance the conditions at the other faces, author derives chief properties of what he calls the "thickness mode" of propagation (since its behavior is dominated by the thickness dimension). The thickness mode corresponding to an infinite phase velocity (cut-off frequency) is found to occur at that frequency for which the half wave length in an infinitely wide rod would be $2d$. The thickness mode cut-off frequency is calculated and good agreement found with earlier measurements [title source, **20,** p. 833, 1948]. A comparison between calculated and experimental values of phase velocity dispersion as a function of a/d shows that there is good agreement as long as d is several times as great as a.

I. N. Sneddon, England

Elasticity Theory

(See also Revs. 1467, 1488, 1499, 1504, 1506, 1508, 1515, 1518)

1473. Sen, Bibhutibhusan, Note on the stresses produced by nuclei of thermo-elastic strain in a semi-infinite elastic solid, *Quart. appl. Math.* **8, 4,** 365–369, Jan. 1951.

Work of Goodier [*Phil. Mag.* **23,** p. 1017, 1937] on the state of thermoelastic equations has been extended by author to obtain displacements and stresses in an isotropic semi-infinite elastic solid having a nucleus of thermoelastic strain at any specific point below the plane boundary.

S. K. Ghaswala, India

1474. Mindlin, Raymond D., and Cheng, David H., Nuclei of strain in the semi-infinite solid, *J. appl. Phys.* **21, 9,** 926–930, Sept. 1950.

By differentiating and integrating the solution for a single force in a semi-infinite solid, Galerkin vector stress functions are derived for forty nuclei of strain. It is pointed out that results may be employed profitably in a number of problems.

B. R. Seth, India

1475. Erim, Kerim, On Saint Venant's principle (in French), *Proc. seventh int. Congr. appl. Mech.* **I,** 28–32, 1948.

R. von Mises ["On Saint Venant's principles," *Bull. Amer. math. Soc.* **51, 1945**] has formulated with precision the title principle and has demonstrated that it is only valid when supplementary conditions exist. Author examines two elementary problems of plane elasticity and determines the supplementary conditions essential to validity of the principle in question, as follows: (1) Half plane $y \geqq 0$ submitted in neighborhood of $y = 0, x = 0$, to a system of forces $P^{(\nu)}$ parallel to the y-axis. (2) Same disk when applied forces have non-null components along the x-axis.

In the first case, these conditions should be $\Sigma P^{(\nu)} = 0, \Sigma P^{(\nu)} \xi_\nu = 0$, i.e., the conditions of equilibrium of the forces $P^{(\nu)}$. In the second case, there should be first $\Sigma P_x{}^{(\nu)} = \Sigma P_y{}^{(\nu)} = \Sigma P_y{}^{(\nu)} \xi_\nu = 0$, i.e., the conditions of equilibrium of the $P^{(\nu)}$, and then also $\Sigma P_x{}^{(\nu)} \Sigma_\nu = 0$, which is a supplementary condition, ξ_ν representing the abscissa of application of force $P^{(\nu)}$.

Maurice Bricas, Greece

1476. Satoh, Tunezo, A new analytical method for plane stress problems, *Jap. J. Math.* **19,** 233–262, 1948.

The theory of Laplace's equation $\Delta w = w_{xx} + w_{yy} = 0$, where $w(x, y)$ is a real-valued function, is related in a known manner to the theory of functions $f(x + iy) = u(x, y) + iv(x, y)$ of the complex variable $x + iy$, where $i^2 + 1 = 0$, $x, y, u(x, y), v(x, y)$ are real, and the functions u and v satisfy the Cauchy-Riemann equations $u_x = v_y, v_x = -u_y$. L. Sobrero [*Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (6) **19,** 77–82, 135–140, 479–483, 1934] has shown that the theory of the biharmonic equation $\Delta\Delta w = w_{xxxx} + 2w_{xxyy} + w_{yyyy} = 0$, where $w(x, y)$ is a real-valued function, is similarly related to the theory of functions

$$f(x + jy) = a(x, y) + jb(x, y) + j^2c(x, y) + j^3d(x, y)$$

of the hypercomplex variable $x + jy$, where $(j^2 + 1)^2 = 0$, $x, y, a(x, y), b(x, y), c(x, y), d(x, y)$ are real, and the functions a, b, c, d satisfy the Cauchy-Riemann equations $a_x = b_y, b_x = c_y - 2a_y$ $c_x = d_y, d_x = -a_y$. Sobrero also showed that the theory of the biharmonic equation is related to the theory of analytic (in a certain sense) functions $f(\alpha + \tilde{\omega}\beta)$ of the complex variable $\alpha + \tilde{\omega}\beta$, where α and β are ordinary complex numbers, and $\tilde{\omega}^2 = 0$. Extending Sobrero's result, the present author shows that the theory of the equation

$$\lambda_1^2\lambda_2^2 w_{xxxx} + (\lambda_1^2 + \lambda_2^2) w_{xxyy} + w_{yyyy} = 0 \qquad [*]$$

λ_1 and λ_2 being real constants, $w(x, y)$ real-valued, is related to the theory of functions

$$f(x + jy) = a(x, y) + jb(x, y) + j^2c(x, y) + j^3d(x, y)$$

where $j^4 + (\lambda_1^2 + \lambda_2^2)j^2 + 1 = 0$, $x, y, a(x, y), b(x, y), c(x, y), d(x, y)$ are real, and the functions a, b, c, d satisfy the Cauchy-Riemann equations $a_x = b_y, b_x = c_y - (\lambda_1^2 + \lambda_2^2)(\lambda_1^2\lambda_2^2)^{-1}a_y$, $c_x = d_y, d_x = -(\lambda_1^2\lambda_2^2)^{-1}a_y$. It is also shown that the theory of equation [*] is related to the theory of analytic (in a certain sense) functions $f(\alpha + \tilde{\omega}\beta)$, where α and β are ordinary complex numbers, and $\tilde{\omega}^2 = \omega\tilde{\omega}$, for a certain complex number ω. All these considerations are closely related to those of Matsumoto [op. cit., 441–482, 1947] and present paper seems to be the paper by Satoh referred to by that author.

Courtesy of Mathematical Reviews J. B. Diaz, USA

1477. Jung, H., On an application of the Fourier transformation in the theory of elasticity (in German), *Ing.-Arch.* **18,** 4, 263–271, 1950.

The Fourier integral solution of the plane biharmonic equation is derived and the corresponding expressions for stresses and displacements are obtained. These results are used to discuss problems of the half plane and the infinite strip subjected to normal and tangential tractions. Mixed boundary-value problems and those in which boundary displacements are given are not treated. H. G. Hopkins [AMR **3,** Rev. 1874] has discussed similar problems more exhaustively and has also obtained some numerical results.

B. R. Seth, India

1478. Freiberger, W., On the solution of the equilibrium equations of elasticity in general curvilinear coordinates, *Austral. J. sci. Res. Ser. A*, **2,** 4, 483–492, Dec. 1949.

A system of stress functions consisting of a harmonic vector and a harmonic scalar potential is introduced into equations of equilibrium of an isotropic elastic body in arbitrary curvilinear coordinates and tensor form. Components of displacement and of stress tensor are expressed in terms of these potentials and it is shown that total number of independent components of the potentials is reducible to three. Tensor equations are then specialized to orthogonal curvilinear coordinates and expressions derived for the "physical" components of displacement and stress in terms of stress functions. Reference is made to relation between present approach and previous ones. From author's summary

1479. Levi, Franco, Direct study of elastic equilibria in presence of non-compatible deformations (in French), *C. R. Acad. Sci. Paris* **231**, 3, 209–211, July 1950.

The author considers a plane elastic system subject to initial stress. It was shown by Colonnetti [*Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (5) **27**$_2$, 267–270, 331–335, 1918] that the corresponding strains $\bar{e}_{ij}$ (which need not satisfy the conditions of compatibility) must satisfy a certain variational principle. By introducing the Airy stress function F and writing out the Euler equation of this variation, the author obtains the differential equation $\nabla^4 F + E\,(\bar{e}_{yy,xx} - 2\bar{e}_{xy,xy} + \bar{e}_{xx,yy}) = 0$, where E is Young's modulus. C. Truesdell, USA

1480. Kammerer, Albert, Variations of Hooke's law and the region of stability (in French), *C. R. Acad. Sci. Paris* **231**, 15, 681–683, Oct. 1950.

Paper deals with homogeneous elastic body for which principal stresses σ_i are quadratic functions of principal strains e_i. Jacobian of transformation from σ_i to e_i is Δ. Stability condition is then $\Delta = 0$. In σ_i space, this condition defines a surface which divides space into several regions, one of which gives stability. Geometry of this surface is considered in special cases.

G. E. Hay, USA

1481. Poritsky, H., Stresses and deflections of cylindrical bodies in contact with application to contact of gears and of locomotive wheels, *J. appl. Mech.* **17,** 2, 191–201, June 1950.

Author gives a novel derivation of the formulas for two-dimensional stresses and displacements of two cylindrical bodies in contact under normal and tangential loads. Contours of principal stress difference are plotted for the normal Hertz distribution, a tangential distribution proportional to it, and a superposition of the two. The first and last of these are in a form suitable for comparison with photoelastic fringe patterns. Contact between gear teeth, rolling cylinders, and a wheel rolling on a track are discussed, including the phenomenon of locking of part of the contact surface. It is shown that, when the elastic constants of the two bodies are not the same, the presence of tangential loading requires a modification of the Hertz distribution.

R. D. Mindlin, USA

1482. Joseph, J. A., and Brock, J. S., The stresses around a small opening in a beam subjected to pure bending, *J. appl. Mech.* **17,** 4, 353–358, Dec. 1950.

Using the complex variable method for plane stress, authors derive solutions for elastic stress distribution around small opening or hole of ovaloid shape in web of beam subjected to pure bending. By varying constants in mapping function, various shapes of opening may be obtained, including the following: Ovaloid hole with axis vertical or horizontal; square hole with rounded corners and side vertical or diagonal vertical. Calculated tangential stress distributions around edge of hole for several cases are also shown. A. M. Wahl, USA

1483. Weinstein, Alexander, New methods for the estimation of torsional rigidity, *Proc. Symp. appl. Math.* **3,** 141–161, 1950.

Author observes fact that since stiffness S of a prismatic bar in torsion equals difference between a polar moment of inertia of the cross section and the Dirichlet integral D, one can reduce the problem of upper and lower bounds for S to same problem for D. He then develops a general method by which bounds for D or for any other positive quadratic functional can be obtained using a procedure based on Schwarz's inequality and Green's formula. The analysis is presented for multiply connected domains.

Both the Raleigh-Ritz method of upper bounds and the Trefftz method of lower bounds are discussed, with a special comment on the limited type of problem to which Trefftz method applies. An interesting comment is also made on a vector interpretation of these two methods. Consideration is given to the improvement of bounds using method of arbitrary parameters and to convergence of bounds if the number of parameters is gradually increased. As a simple application of the general theory an inequality for the torsional rigidity of a hollow square is derived.

Second half of paper is devoted to the method of symmetrization which provides inequalities giving upper bounds for torsional rigidity. The method results from a study of an extremal property in an isoperimetric problem. W. H. Hoppmann, II, USA

1484. Wolf, H., The propagation of torsional plastic waves in circular cylindrical tubes and shafts, *Grad. Div. appl. Math. Brown Univ., Tech. Rep.* A11-50, 41 pp., 1949.

In the reduction of the three-dimensional equations of small motions of a continuum to special forms applicable to longitudinal and transverse motions of bars, it is customary to neglect terms which are important for wave lengths short in comparison with lateral dimensions of the bar. In the case of torsional motions of a bar or tube of circular cross section it is not necessary to neglect such terms in order to arrive at a single, second-order, partial differential equation of motion. Solutions of this equation may be expected to conform more closely with impact experiments than solutions of the corresponding equations for longitudinal and transverse motions. Author considers case of torsional motions and shows that, whereas for an elastic medium the final equation is linear and contains only one independent space variable, for a plastic medium the equation is nonlinear and contains two independent space variables. Nevertheless, he is able to obtain useful solutions, by a method of successive approximations, for a reasonable deformation law of plasticity and appropriate boundary conditions. R. D. Mindlin, USA

1485. Skudrzyk, E., Internal friction and material losses of solid bodies. I. General theory (in German), *Öst. Ing.-Arch.* **3,** 4, 356–373, 1949.

This paper attempts to treat elastic aftereffects by the common expedient of superposing ordinary elastic and viscous effects [cf., e.g., Weissenberg, *Abh. Preuss. Akad. Wiss. Phys.-Math. Kl.* 1931, no. 2]. Author's theory is tacitly limited to infinitesimal displacement gradients, since he does not distinguish between Eulerian and Lagrangian coordinates and since he employs local rather than material time derivatives. He proposes the familiar expression $t^i{}_j = f(\partial/\partial t)\xi^k{}_{,k}\delta^i{}_j + g(\partial/\partial t)(\xi^i{}_{,j} + \delta_{j,}{}^i)$, where $t^i{}_j$ is the stress tensor, ξ^i is the displacement vector, and f and g are power series. He notes that (questions of convergence being neglected) in a harmonic oscillation f and g reduce to complex power series in the frequency, so that the response of the material may then be characterized by four real functions of frequency. He discusses the definition of elastic moduli and damping coefficients appropriate to various special cases in terms of these functions. A second part is to follow. C. Truesdell, USA

Experimental Stress Analysis

(See also Revs. 1562, 1616)

1486. **Lee, George Hamor, An introduction to experimental stress analysis,** New York, John Wiley & Sons, Inc., 1950, xiv + 319 pp. $5.50.

The reader is first given a background developed by an adequate discussion of the theory used in stress analysis. This includes an introduction to the theory of elasticity, stress-strain relations, structural analysis, similitude and dimensional analysis. These subjects are treated with considerable finesse, including just enough material to hold continuity. At the same time sufficient detail is included so that the reader can establish a background sufficient for him to extract all the implications of the various experimental methods and techniques presented in the rest of the book.

All the important and commonly used methods and techniques of experimental stress analysis are described and their utility discussed in the bulk of the book. This includes such topics as mechanical, optical, and acoustical strain gages, electrical gages with descriptions of their applications and limitations, measurement of static, oscillatory and transient strains, photoelasticity, analogies, brittle coatings and models, photogrids, Begg's deformeter, calibration, and errors. These descriptions are presented in a clear, readily understandable style.

The book develops the subject in a concise, interesting, and logical manner without losing any of the over-all perspective so essential in a book of this type. Its value is enhanced by the carefully selected bibliographies at the end of each chapter.

C. O. Dohrenwend, USA

1487. **Favre, H., and Gilg, B., On a purely optical method of directly measuring the moments in thin plates under bending** (in French), *Schweiz. Bauztg.* **68,** 19, 20, 253–257, 265–267, May 1950.

When plane-polarized light is passed through a bent plate of transparent, homogeneous material, the photoelastic effects produced on one side of the neutral surface are offset by the balancing stresses on the other side. Authors overcome this difficulty by constructing the plate of two cemented layers, one of optically stress-sensitive CR39, the other of glass. [A like suggestion is due to Mindlin, *J. appl. Mech.* **8,** p. A–188, 1941.] It is shown that, subject to the limitations of ordinary plate theory and provided that the two layers have essentially the same Poisson ratio, the directions of polarization coincide with the directions of principal moment and that relative retardation is proportional to $M_1 - M_2$, the difference between principal moments. Analogous to two-dimensional photoelasticity, absolute retardations $\delta_1 = AM_1 + BM_2$, $\delta_2 = CM_1 + DM_2$. Material constants $A, \ldots, D$ are found by applying known moments to test pieces.

Practical application of the foregoing principles is fully described. Authors measure δ_1, δ_2 directly using a Mach-Zehnder interferometer. Conventional photoelastic techniques may also be used. The experimental accuracy indicates that technique can be employed to good advantage in determining the state of moments in plates of irregular contour.

L. E. Goodman, USA

1488. **Gorton, R. E., and Pratt, R. W., Strain measurements on rotating parts,** *Trans. Soc. auto. Engrs.* **3,** 4, 540–556, Oct. 1949.

Application of bonded-resistance strain gage to measurement of strain on such rotating parts as crankshaft, fan blades, and supercharger impellers is discussed. Applications and techniques covered are said to have served well on reciprocating engine problems and to have been adapted readily to turbine engine work. Special techniques are reported for measurement of centrifugal strain in rotors, and for vibratory strain measurements in high-temperature turbine blades.

From authors' summary

1489. **Riparbelli, C., A method for the determination of initial stresses,** *Proc. Soc. exp. Stress Anal.* **8,** 1, 173–196, 1950.

In order to determine residual stress in the surface of a structure, electric strain gages are applied, a hole is drilled in the vicinity, and the variation of strain measured. As adequate sensitivity may be obtained with holes of only $^1/_8$-in. diam, method can be considered to be nondestructive with respect to large statically loaded structures. To determine a uniaxial stress, four gages around the hole or only two gages are used. In the most general case of a uniform plane-stress distribution, three independent parameters must be read by means of rosettes at 45° or 60°. The theoretical and experimental calibration for plane-stress distribution are described. Mathematical and graphical solutions (e.g., Hansen's construction) for the rosette problem in plane-stress distribution—especially the radial point gage rosette with uniform initial stress—are given. The stress can be measured in the described way in a few seconds—the time necessary to drill the hole. Advantage of the method is evident, especially if analyzing the stress of large structures (example is given for hyperstatic arch bridge) or residual stresses in welded plates (example is given for the stress distribution in two welded plates). The determination of stress in bent plates of small thickness is also considered. Many examples and diagrams are included.

Ferdinand Budinsky, Czechoslovakia

1490. **Ramanchandran, G. N., Photo-elasticity of diamond,** *Proc. Indian Acad. Sci. Sec. A,* **32,** 3, 170–173, Sept. 1950.

The photoelastic constants of diamonds have been redetermined and are found to be $q_{11} = -5{\cdot}0_5$, $q_{12} = +2{\cdot}1_5$, $q_{44} = -2{\cdot}8 \times 10^{-14}$ cm² dyne^{-1} and $p_{11} = -0{\cdot}31$, $p_{12} = +0{\cdot}09$, $p_{44} = -0{\cdot}12$. These lead to a decrease in refractive index when diamond is subjected to a hydrostatic pressure.

From author's summary

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 1459, 1481, 1482, 1546, 1619, 1685)

1491. **Pardue, T. E., and Vigness, Irwin, Properties of thin-walled curved tubes of short-bend radius,** *Trans. Amer. Soc. mech. Engrs.* **73,** 1, 77–87, Jan. 1951.

Presents high stress intensification and flexibility factors measured and computed for sharply curved thin-walled pipes. Theory includes previously neglected higher-order terms. Tests show stress and flexibility are sensitive to restraints at bend ends. Straight pipes at bend ends diminish this sensitivity. Discussers Gross and Ford mention British pressure tests on curved tubes but include no data.

Herbert Becker, USA

1492. **Coepijn, W. C., Adjusting factors for symmetrically bent or buckled bars** (in Dutch), *Ingenieur* **62,** 18, 13–20, May 1950.

Paper gives a short explanation of how to calculate the main data needed in using the Cross method for bar constructions containing symmetrically bent or buckled bars.

From author's summary

1493. **Demoulas, A. D., Deformation of the cross sections of beams in bending** (in Greek), *Tech. Chronika, Athens* **27,** 308, 91–97, Feb. 1950.

Paper considers stresses at each cross section of a beam sub-

jected to an eccentric load. Stresses are computed by transforming the given cross section into a related one through the transformation $x' = ax$ and $y' = by$. The new coordinates of the point of load application are given by $X' = kX$, $X' = aX$, and $Y' = bY$, where k is the ratio of original to transformed cross-sectional area. This new method may be used advantageously to compute stresses on cross sections whose two axes of symmetry are oblique by transforming these cross sections into ones with orthogonal axes of symmetry. Dimitri Kececioglu, USA

1494. **Swida, W., The deformation energy theorems applied to the elasto-plastic state** (in German), *Ing.-Arch.* **16**, 221–230, 1948.

Author is concerned with energy theorems associated with elastic-plastic bending of a beam consisting of a nonstrain-hardening material which obeys Hooke's law up to the yield point. Elastic unloading is assumed. From the simple graph of concentrated transverse load P versus displacement δ of its point of application, he deduces following relations: $dU/d\delta = P$, $dU_k/dP = \delta$, $dU_e/dP = \delta_e$ and $dU_r/dP = \delta_r$, where U is work done, U_e is elastic energy (recoverable upon unloading), $U_k = U_e + U_r$ is what author calls "potential energy," U_r is the nonrecoverable part of U_k and is called the residual or permanent energy of deformation, δ_r is permanent displacement, δ_e is elastic displacement. Author notes that the first two relations are not new but fails to identify the quantity U_k as being the familiar "complementary energy" of F. Engesser [*Z. Architektur u. Ingenieurwesen Hannover* **35**, 734–742, 1889]. The last two relations are new. Author applies some of these relations to examples involving statically determinate beams and derives the principle of virtual displacements for use in statically indeterminate problems. Finally, he notes that the Maxwell-Betti reciprocity theorem is no longer valid in the elastic-plastic domain. All of these things are well known in more general terms.

H. J. Greenberg, USA

1495. **Panayotounakou, E., Statically indeterminate helical beams. The fixed-ends helical beam. The helical beam with a fixed end and a spherical joint** (in Greek), *Tech. Chronika, Athens* **27**, 313–314, 401–409, July-Aug. 1950.

In this paper two more significant applications of the theory developed in author's two previous papers are given [AMR **3**, Revs. 445, 446]. Dimitri Kececioglu, USA

1496. **Cavadias, G. S., A new method of computing continuous beams** (in Greek), *Tech. Chronika, Athens* **27**, 313–314, 410–419, July-Aug. 1950.

The new method consists of using a series of double-span beams as the basic principal system. Computing these beams is made easy by the introduction of three prepared nomograms. This new method presents to lesser advantage the moment distribution principle than do the methods of H. Cross and E. Shepley. An example illustrates the application of the new method.

Dimitri Kececioglu, USA

1497. **Stevenson, A. C., The centre of flexure of a hollow shaft,** *Proc. Lond. math. Soc.* (2) **50**, 536–549, 1949.

Paper may be taken as supplementing two papers of B. R. Seth in which bending of a hollow shaft with an eccentric hole is discussed [cf. *Proc. Indian Acad. Sci. Sec. A*, **4**, 531–541, 1936; **5**, 23–31, 1937]. In a previous paper, author [*Trans. roy. Soc. Lond. Ser. A*, **237**, 161–229, 1938] defined the center of flexure as a load point in the section of a cantilever beam such that the mean twist taken over the section is zero and obtained results for a large number of sections with help of some canonical flexure functions and moment integrals. Using this method he now solves the flexure problem for a hollow shaft and obtains the position of the center of flexure. When the circular boundaries of the shaft are in contact, results are expressed in terms of trigamma and tetragamma functions, and numerical values are obtained for the associated twist and center of flexure.

B. R. Seth, India

1498. **Pode, Leonard, An experimental investigation of the hydrodynamic forces on stranded cables,** *David W. Taylor Mod. Basin Rep.* 713, 17 pp., May 1950.

Experimental evidence verifies the sine-square law for the normal component of the hydrodynamic force acting on stranded cables. The side component appears to be proportional to the sine of the angle of inclination of the cable to the stream for very small angles, attaining a maximum value at an angle that depends upon the size and construction of the cable.

From author's summary

1499. **Freiberger, W., and Smith, R. C. T., The uniform flexure of an incomplete tore,** *Austral. J. sci. Res. Ser. A*, 2, 4, 469–482, Dec. 1949.

Authors discuss flexure of an incomplete tore in the plane of its circular center line. They reduce problem to determination of two harmonic functions, subject to boundary conditions on the surface of tore which involve the first two derivatives of the functions. They point out the relation of this solution to the general solution of three-dimensional elasticity problems. Special case of a narrow rectangular cross section is solved exactly.

From authors' summary

1500. **Morley, Arthur, Torsional stress in close-coiled helical springs,** *Engineering* **169, 170;** 4388, 4401, 4402, 4406; 231–232, 630, 658; 15–16; Mar.; June; July 1950.

Sopwith, D. G., Torsional stress in close-coiled helical spring, *Engineering* **169**, 4392, 4401; p. 360, p. 630; Mar.; June 1950.

Adams, L. E., Torsional stress in close-coiled helical springs, *Engineering* **169, 170;** 4404, 4411; p. 708, p. 135; June; Aug. 1950.

Finniecome, J. R., Torsional stress in close-coiled helical springs, *Engineering* **170**, 4406, p. 16, July 1950.

It is known that the shear stress in the wire or bar of a coil spring is higher at the side of its section nearest to the center of the coil, but no exact method for computing its maximum value has as yet been developed. A number of close approximations have been formulated by various investigators, and Morley in first paper derives three additional ones, one of which leads to the well-known Wahl formula, while the other two give slightly differing maximum stress. Like all earlier investigators, Morley establishes a curvature correction factor, which, with the maximum shear stress occurring in a straight bar, gives an increased value, due to the curvature.

In the discussion by D. G. Sopwith, the corresponding factors of ten other investigators are given. Sopwith also states that complete information on the stresses in round and rectangular bars will be given in the British Standard "Guide to the specification of helical compression springs," to be issued shortly.

With one exception (Dick), all the investigators arrive at practically the same results, all within the range of unavoidable manufacturing variations. It is, therefore, not possible to determine by experiment which one is the more exact.

O. R. Wikander, USA

1501. **Blanas, S., On the beam with a variable cross section** (in Greek), *Tech. Chronika, Athens* **27**, 312, 341–351, June 1950.

An improvement to the existing methods of analysis of indeter-

minate structures is proposed, as applied to beams with a variable cross-sectional moment of inertia. The method consists of, first, determining the support angles; second, calculating the support slopes by a semigraphical method; and last, introducing these results into the conventional continuous beam equations obtained by either the moment-distributon or the three-moment method. A worked-out example illustrates the expediency of this new method of analyzing indeterminate beams.

Dimitri Kececioglu, USA

1502. **Winter, George, Lansing, Warner, and McCalley, R. B., Jr., Performance of laterally loaded channel beams,** *Cornell Univ. Engng. Exp. Sta.*, reprint 33, 39–50, Nov. 1950.

See AMR 3, Rev. 1868.

Plates, Disks, Shells, Membranes

(See also Revs. 1487, 1491, 1534, 1535, 1690, 1719)

1503. **Uflyand, Ya. S., On the solution of the problem of deflection of rectangular and sectorial plates for some boundary conditions** (in Russian), *Doklady Akad. Nauk SSSR* **74**, 3, 437–439, Sept. 1950.

A method is outlined for solving rectangular plates, built-in along one edge and simply supported along opposite edge, by means of trigonometric series and integral equations. Two remaining edges of plate may be built-in or simply supported. Method is also applicable to sectorial plates built-in along one radius and simply supported along the other.

M. Gololobov, Czechoslovakia

1504. **Das, Sisir Chandra, Note on the bending of certain thin elastic plates by concentrated loads,** *Bull. Calcutta math. Soc.* **42**, 2, 89–93, June 1950.

A simple method developed by B. Sen in 1934 [*Indian Physics math. J.* 5, 17] of finding the deflection of elastic plates due to concentrated loads is applied to two cases. The first is a clamped plate bounded by an inverse of an ellipse loaded by a concentrated load at the center. The second is a clamped plate bounded by an elliptic limaçon loaded by a concentrated load at the focus. In both cases the solutions are obtained in closed form.

R. L. Bisplinghoff, USA

1505. **Carrier, G. F., and Shaw, F. S., Some problems in the bending of thin plates,** *Proc. Symp. appl. Math.* **3**, 125–128, 1950.

Paper deals with use of "eigenfunctions" in solving thin plate problems. Two illustrative problems are solved: (1) Sector-shaped plate clamped at circular edge and loaded by a torque at, or near, vertex; and (2) clamped sectorial plate subjected to a uniform load. Bending of plates is considered. Method proposed is approximate, but practical, and saves time.

Cameron M. Smith, USA

1506. **Funk, P., and Berger, E., Bound for the maximum deflection of a uniformly loaded, built-in quadratic plate** (in German), Wien, Franz Deuticke, *Federhofer-Girkmann-Festschrift*, 199–204, 1950.

Consider a uniformly loaded, clamped square plate in its equilibrium position. At station x, y, where the static deflection $u(x, y)$ is to be determined, impose a transverse force P. Let $E(P)$ be the work done by P against the internal forces. By means of the calculus of variations and basic principles of elasticity theory, upper and lower limits for $E(P)$ can be found. These being known, the theorem of Castigliano immediately yields two close bounds for the required deflection $u(x, y)$. These are seen to be in complete agreement with the investigations of Ritz.

Max A. Dengler, USA

1507. **Jaramillo, T. J., Deflections and moments due to a concentrated load on a cantilever plate of infinite length,** *J. appl. Mech.* **17**, 1, 67–72, Mar. 1950.

Author solves the biharmonic equation for deflections in integral form and deduces the bending and twisting moments also in finite form. To evaluate these, a series expansion is obtained by use of appropriate contour integrals. A numerical solution is given, but while it is stated that the series converge rapidly, no convergence proof is given. Results are discussed for boundary conditions in which one edge is clamped and one free. Comparison is made with MacGregor's special case solution, in which the load is at the free edge. Agreement is good.

R. C. Knight, England

1508. **Fridman, M. M., Deflection of a thin isotropic rectangular plate with a welded-in round isotropic disk from another material** (in Russian), *Prikl. Mat. Mekh.* **14**, 429–432, July–Aug. 1950.

Solution applies to case for which diameter of the disk is small compared to dimensions of the plate, and external load is reduced to constant bending moments along the plate boundaries. Author shows that separate solutions for the disk and the plate can be obtained by means of appropriate power series of the complex variable. Boundary conditions at the plate and disk junction involve, in addition to continuity of deflection and its partial derivative with respect to radius, equality of the radial moment and the Kirchhoff requirement [See Timoshenko, S., "Theory of plates and shells," McGraw-Hill, 1940, p. 89] regarding twisting moment and shear force. This last requirement is legitimate, in so far as the solution for the plate is concerned, because of smallness of the disk. Under these conditions, the solution is reduced to four terms. The following particular cases are discussed: (1) Plate is bent by moments uniformly distributed along two opposite sides, while the other two are free. By assuming that modulus of elasticity of the disk is zero or infinity, the previously derived solutions for a plate with a small circular hole or a plate clamped along an inside diameter are obtained. (2) Plate is bent by moments uniformly distributed along all four sides. In this case the twisting moment and the shear force are equal to zero. (3) Plate is loaded by twisting moments uniformly distributed along all four sides. In all three cases, the solution for the disk does not depend on the radius, r, and the maximum moments occur along the plate and disk junction.

D. Rosenthal, USA

1509. **Osgood, W. R., and Joseph, J. A., On the general theory of thin shells,** *J. appl. Mech.* **17**, 4, 396–398, Dec. 1950.

Love's expression for the change of curvature τ in his general theory of thin shells has an unsymmetrical character, and, therefore, is open to suspicion as to its correctness. Authors derive analogical expressions and obtain different equations for τ and for equilibrium which are more accurate than Love's in that they do not require the assumption that deformation of middle surface is inextensional (in latter case their expressions would agree with Love's). In 1930 reviewer noticed a difference between the equations of equilibrium for a cylindrical and spherical shell derived by Meissner (based on general theory of Love) and the analogical equations derived by himself by integrating the equations of equilibrium which hold for a thick shell [Love, Ch. II. 59, p. 90] with respect to the coordinate z, denoting the normal to middle surface. Reviewer considers that proof of correctness of eq. (10) would be the derivation of eq. (9) and especially (10) by integrat-

ing the general equations of equilibrium expressed in curvilinear coordinates which hold for a thick shell with respect to z.

M. Hampl, Czechoslovakia

1510. Hildebrand, F. B., On asymptotic integration in shell theory, *Proc. Symp. appl. Math.* **3,** 53–66, 1950.

Author applies method of asymptotic integration to the system of simultaneous equations, due to E. Meissner, for the equilibrium of thin shells with rotational symmetry, and arrives at somewhat more general results than O. Blumenthal [*Proc. fifth Congr. Math.* **2,** p. 319, 1912], who treated the special case of a spherical shell with constant thickness. Solutions for the homogeneous Meissner equations are given in the form of asymptotic power series with respect to the parameter $1/k$, where k^2 is proportional to shell thickness and to distance to axis of revolution along the normal of the middle surface. The leading term in the corresponding expressions for shearing force, moments, displacements, and angular deflection of the middle surface are given. The inhomogeneous equations are treated with expansions of similar type, the coefficients of which are determined from recurrence formulas. Thus, if it is assumed that the distributed surface loading may vary appreciably only over a distance, comparable with a representative dimension of the shell, it follows that the complete solution may be composed of a "membrane solution" and a solution of the homogeneous equations. Order of magnitude of the stresses and deformations with respect to parameter k is also investigated, permitting interesting conclusions. For example, the membrane solution and membrane moments in general turn out to be negligible as compared with corresponding term in the homogeneous solution. Should, on the other hand, the membrane rotation be of importance, then further terms in the asymptotic developments of the homogeneous solution must be retained.

Folke K. G. Odqvist, Sweden

1511. Tolotti, Carlo, Statics of developable inextensible and elastically flexible surfaces (in Italian), *G. Mat. Battaglini* (4), **3** (79), 1–48, 1950.

The author continues his researches in the theory of elastically flexible but inextensible membranes for which he had derived the general equations [*Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **1,** 369–374, 605–609, 1946; AMR **4,** Rev. 1083]. In the present paper he obtains the general solution of his equations for the case of a developable surface, generalizing the solutions given by Laura [*Ist. Veneto Sci. Lett. Arti. Parte II Cl. Sci. Mat. Nat.* **99,** 339–356, 1940] for Beltrami's equations (in which the membrane is supposed to be without flexural elasticity). He shows that if the deformed configuration Γ of a curve upon the undeformed membrane is described, then the rectifying developable of Γ is an equilibrium configuration for the membrane. As might be expected, however, it is generally not possible to find a solution corresponding to prescribed forces and moments upon the boundary. The author employs a variational principle to obtain suitable boundary conditions in terms of forces, and these he discusses and interprets. Finally, he transforms the equation into another form suitable for attack by the direct method of the calculus of variations, to which he promises to devote a subsequent paper.

C. Truesdell, USA

1512. Gradwell, C. F., Asymmetrical bending of tapered disks, *Aircr. Engng.* **22,** 257, 209–212, July 1950.

The problem of asymmetrical bending of a disk of variable thickness is treated for two variations of boundary conditions: (1) Disk built in at the bore and simply supported at outer circumference; (2) built-in bore, fixed at outer circumference. The twisting couple is applied about a diameter. There is no exact solution for many types of thickness variation. If, however, thickness varies as some power of radius, an exact solution can be found. A number of exact solutions for power type of thickness variation have been worked out.

A β factor, plotted for a family of disk profiles is given. If the disk does not have a hyperbolic contour, an approximation is used. Value of β obtained from disk contour is then used in formulas given to obtain stresses and deflections.

Author indicates that the centrifugal stress system due to rotation of disk may be superimposed on twisting stress system. Reviewer feels that this would result in error since the stiffening effect of the centrifugal force field would not be taken into account.

E. G. Allen, USA

1513. Darevskiĭ, V. M., Action of a concentrated load upon a cylindrical shell (in Russian), *Doklady Akad. Nauk SSSR* (N.S.) **75,** 1, 7–10, Nov. 1950.

Asymptotic formulas are given for the stresses in the neighborhood of concentrated loading of a cylindrical shell. Displacements u, v, w and shearing forces N_1, N_2 are determined in known manner by equations of the form $L_{\nu 1}u + L_{\nu 2}v + L_{\nu 3}w + L_{\nu 4}N_1 + L_{\nu 5}N_2 = P_\nu$ ($\nu = 1, 2, 3, 4, 5$), where $L_{\nu\mu}$ are differential operators in the coordinates on the middle surface of the thin shell and the P_ν proportional to the loading components. After solving the said equations, displacements and both normal and tangential forces are given, as well as the bending and twisting moments corresponding to the different components of loading.

H. Schlechtweg, Germany

1514. Marshall, W. T., The application of relaxation methods to freely-supported flat slabs, *Engineering* **170,** 4417, 239–242, Sept. 1950.

Author gives a detailed derivation of the difference equations for the Lagrange plate equation and for the moments. Relaxation methods are employed to solve problem of simply supported rectangular plates with concentrated loads. Two examples are solved by repeated use of the harmonic difference operator.

D. L. Holl, USA

1515. Friedrichs, K. O., Kirchhoff's boundary conditions and the edge effect for elastic plates, *Proc. Symp. appl. Math.* **3,** 117–124, 1950.

Author indicates a systematic derivation of Kirchhoff's boundary conditions for free edge of elastic plate by systematic expansion of the stresses and displacements with respect to powers of plate thickness. In this way, not only are Poisson's three conditions satisfied, but stresses at the free edge vanish. Method considers stresses as made up of two parts, "interior stresses" and quickly varying "excess stresses" near the edge. Numerical values for excess quantities at edge in terms of interior quantities are given.

S. Levy, USA

1516. Petersen, James P., Experimental investigation of stiffened circular cylinders subjected to combined torsion and compression, *Nat. adv. Comm. Aero. tech. Note* 2188, 16 pp., Sept. 1950.

Five Z-stiffened cylinders were tested to failure by local stringer crippling under various combinations of torsion and compression. Experimental results were found to be in accord with available theory on sheet buckling [AMR **1,** Revs. 441, 442] and on stiffener stresses and maximum strength of stiffened cylinders under torsion [title source, 1481]. Semi-empirical method is developed for estimating stiffener stresses in cylinder under combined loads and an interaction curve is given for the strength of the cylinder.

S. B. Batdorf, USA

1517. **Dow, Norris F., and Hickman, William A., Effect of variation in rivet diameter and pitch on the average stress at maximum load for 24S-T3 and 75S-T6 aluminum-alloy, flat, Z-stiffened panels that fail by local instability,** *Nat. adv. Comm. Aero. tech. Note* **2139**, 24 pp., July 1950.

Previously reported experimental tests by authors are correlated by single curve whose abscissa is ratio of square of rivet pitch divided by rivet diameter and total thickness (sum of skin and stiffener thickness), and whose ordinate is ratio of average stress at maximum load to optimum average stress at maximum load (obtained by most favorable combination of rivet diameter and rivet pitch). Curve applies to panel of 24S-T3 aluminum alloy.

Procedure is adopted to panel made of higher strength material, 75S-T6. Single lower limit curve for both materials is constructed. It enables replacing 24S-T3 panel with 75S-T6 panel of same cross section, same or greater load-carrying capacity and modified rivet pitch. The correlation is limited to the range of panels tested. Scatter of test results increases for weaker riveting. Caution should be exercised in using curves when details of construction exceed circumstances under which investigations were conducted.

Use of phrase "potential strength" for the optimum average stress is not desirable in reviewer's opinion. H. A. Lang, USA

1518. **Handelman, G. H., Torsion of thin-walled closed cylinders beyond the elastic limit,** *J. aero. Sci.* **17**, 8, 499–507, 518, Aug. 1950.

Elastic theory of torsion (of Saint Venant type) in thin-walled closed cylinders is extended beyond elastic limit. From assumptions on wall thickness, treatment involves a single shear stress and a single associated shear strain. Formulas for shear distribution of stress, strain, and warping function are developed. Formula for torque vs. angle of twist is derived and problem reduced to equivalent elastic tube by introducing an appropriate artificial thickness. Single-cell and multi-cell section, both of piecewise constant wall thickness, illustrate in detail the numerical-graphical procedures involved. H. A. Lang, USA

1519. **Kostyuk, A. G., On the equilibrium of an annular plate with a power law of hardening** (in Russian), *Prikl. Mat. Mekh.* **14**, 3, 319–320, May–June 1950.

Stress distribution in a thin annular plate is determined under following assumptions: Thickness is given by $h = h_0\rho^{-\lambda}$, where ρ is the normalized radius and λ is a constant. Radial stress distribution is prescribed at the circular edges, stress normal to plate is set equal to zero, and the stress distribution across the thickness is assumed uniform. Hardening law is given by $\sigma_i = N\epsilon_i^{\mu}$ where σ_i and ϵ_i are the stress and strain intensities, respectively, N is a constant, and μ is a parameter in the range $0 \leq \mu \leq 1$. A graph presents the normalized radial and tangential stress distributions in the plate. Solution is next modified to give bending moment distribution in a plate subjected to uniformly distributed moments at the boundary.

Courtesy of Mathematical Reviews H. I. Ansoff, USA

1520. **Suhara, Toyotarô, On the stresses in a circular disc of variable thickness, the coefficients of elasticity and the thermal expansion of the material being taken as functions of temperature** (in Japanese), *Proc. Fac. Engng. Keiogijuku Univ.* **1**, 43–46, 1948.

Stresses due to the rotation and the thermal expansion of a thin circular disk of variable thickness z, which is expressed by $z = z_0(1 - ar^{\nu})^k$, are analyzed for given temperature distributions, $T = T_0(1 + br^{\nu})$. The coefficient of thermal expansion α and that of elasticity are taken as functions of temperature, or as functions of the distance r from the axis of rotation, such as $\alpha = \alpha_0(1 + b'r^{\nu})^h$ and $E = E_0(1 - cr^{\nu})^i$.

From author's summary

Buckling Problems

(See also Rev. 1690)

1521. **Duberg, John E., and Wilder, Thomas W., Column behavior in the plastic stress range,** *J. aero. Sci.* **17**, 6, 323–327, June 1950.

Theoretical behavior of a straight column loaded in compression is discussed. It is assumed that column buckles plastically before Euler (elastic buckling) load is reached. The straight column is considered as the limiting case of a bent column, and it is shown that for linear strain-hardening buckling must occur at the tangent-modulus load introduced by Shanley [AMR **1**, Rev. 72]. Results are extended to case of nonlinear strain-hardening. Details are to be given in a future paper.

An ingenious model, different from Shanley's, is used for linear case. Authors claim that unless the straight column is considered as a limiting bent column, the problem is indeterminate. This difficulty can also be resolved by carefully defining plastic buckling [AMR **1**, Rev. **1203**].

For nonlinear strain-hardening, authors claim that the buckling load is higher than the tangent-modulus load. However, they do not define the tangent modulus for this case.

P. G. Hodge, Jr., USA

1522. **Cicala, P., Column buckling in the elastoplastic range,** *J. aero. Sci.* **17**, 8, 508–512, Aug. 1950.

An extension of Shanley's theory of plastic buckling of columns is presented. A pair of integrodifferential equations are derived for a column of constant H-shaped section, neglecting curvature of the stress-strain diagram. The derivation takes account of the fact that the stress at which strain reversal occurs is different at different sections along the length of the column. A method of solving the equation by a step-by-step procedure is used to obtain numerical results. The midsection flange loads, thus computed, give qualitative confirmation to results obtained by Shanley on an idealized semirigid strut. The lateral deflections and maximum stress increase rapidly as the tangent modulus load is exceeded.

Author makes use of Shanley's semirigid scheme to investigate effect of initial eccentricity and finds that strain reversal occurs in one flange at a load less than the tangent-modulus buckling load. When effect of curvature of stress-strain diagram is also considered, it is found that maximum load resisted by the column is less than the reduced-modulus buckling load by an amount which increases with increase in curvature of the stress-strain diagram. S. Levy, USA

1523. **Thomson, W. T., Critical load of columns of varying cross section,** *J. appl. Mech.* **17**, 2, 132–134, June 1950.

Paper derives formulas for computing critical loads for pin-ended struts consisting of a finite number of discrete lengths having constant flexural rigidity. The necessary deflected form for each section is expressed in terms of its end deflections and an iteration formula for the ratio of deflection to slope at successive section boundaries is derived. In simple cases this yields a transcendental equation for the critical load, but in more complex cases forms the basis for a "trial and error" solution. Experiments upon struts with two or three segments, which confirm the theory, are reported. W. S. Hemp, England

1524. Rasmussen, B. H., Stability of columns with variable moment of inertia (in Danish), *Frandsen Anniv. Vol. Lab. Bygn. Tekn. Medd.*, no. 1, 100–110, 1950.

Author works out in detail the method for study of column stability which was suggested by P. M. Frandsen in his books "Bygningsstatik I" (Theory of structures, vol. I), Copenhagen, 1948, and "Elasticitsteori," Copenhagen, 1948. In present procedure of computation, column is divided into parts of equal length and matrix calculus is used. P. Neményi, USA

1525. Shanley, F. R., Applied column theory, *Proc. Amer. Soc. civ. Engrs.* **75**, 759–788, June 1949.

Euler did not limit his famous formula to elastic buckling of columns but mentioned the possibility of extending it to plastic buckling by a suitable interpretation of the modulus E. In 1889, Engesser showed that there is good agreement between Euler's formula and test results if the secant modulus is used instead of E. Von Kármán confirmed this, and proposed to use Euler's formula with the secant modulus for the beginning of buckling, and a double modulus for the upper limit of the load as bending increases. For the structural engineer, the beginning of buckling is the extreme limit which he is not allowed to pass in any case. Author carries out tests with various materials and boundary conditions, which show that the Euler-Engesser formula is very useful. Following von Kármán's example, the curves of the modulus E are plotted as a function of stress for different materials. Other representations of the tangent modulus which have been confirmed by tests are discussed, especially the relation between modulus and stress in the nondimensional representation. The influence of the material, initial eccentricity, and end fixity on buckling is also thoroughly discussed. Finally, buckling is treated from the viewpoint of weight deduction.

Ludwig Föppl, Germany

1526. Belluzzi, Odone, Contribution to the study of buckling of columns (in Italian), *G. Gen. civ.* **88**, 2, 79–83, Feb. 1950.

Various formulas for approximate calculation of the elastic buckling load of columns with constant and varying cross section are given. The more exact group of these formulas is derived by energy methods or work equations, while the second group represents mean value expressions and is hence less exact. Comparison of calculational results from formulas of different degrees of exactness permits evaluation of the exactitude of the chosen solution curve. Examples, numerical calculations, and exactitude comparisons are included. F. Stüssi, Switzerland

1527. Selberg, A., Calculation of steel columns (in Danish), *Bygnstat. Medd.*, no. 4, 79–107, 1949.

Paper deals with steel columns loaded with concentric or eccentric axial forces. The buckling of built-up columns, such as framed columns and latticed columns, are treated and calculation formulas given which are found to be very accurate. However, as they are too laborious for common practice, simple equations are introduced. From author's summary

1528. Rambøll, B. J., Elastically supported columns (in Danish), *Bygnstat. Medd.*, no. 3, 63–78, 1949.

A method of approximation is developed, by means of which one may very quickly—even for a considerable number of supports—determine number of half waves, as well as critical load, corresponding to a given rigidity of intermediate supports. As an example, lateral stability of the upper chord of a low-truss bridge is considered, and a formula is evolved for determining required rigidity of vertical members.

From author's summary

1529. Gregory, R. W., The Euler buckling load of a strut with crossed pins, *Engineering* **170**, 4410, p. 112, Aug. 1950.

A strut with crossed pins at its ends is twisted so that the pins become parallel to each other; then strut is subjected to an axial load. The strut under these conditions is investigated. Author used incorrect end conditions, and obtained a buckling load higher than the Euler load of the column.

G. V. R. Rao, USA

1530. Muckle, W., Resistance to buckling of light-alloy plates, *Trans. N.E. Cst. Instn. Engrs. Shipb.* **64**, 6, 223–272, Apr. 1948.

Paper gives an account of some experimental work which has been carried out on buckling of light-alloy plates, and which is related to existing experimental work for steel. Possible modes of failure of large stiffened sheets of plating in compression are considered. Some general conclusions are drawn with regard to application of results to ship construction. [See also AMR 1, Rev. 1111.] From author's summary

1531. Heimerl, George J., and Roberts, William M., Determination of plate compressive strengths at elevated temperatures, *Nat. adv. Comm. Aero. Rep.* 960, 6 pp., 1950.

See AMR 2, Rev. 600.

1532. Winter, G., Performance of compression plates as parts of structural members, *Cornell Univ. Engng. Exp. Sta.* reprint 33, 51–57, Nov. 1950.

See AMR 3, Rev. 2261.

1533. Houbolt, John C., and Stowell, Elbridge, Z., Critical stress of plate columns, *Nat. adv. Comm. Aero. tech. Note* 2163, 16 pp., Aug. 1950.

An exact solution is obtained for a plate of dimensions b, L, hinged along sides b, free along sides L, and compressed in the L direction. The buckling stress is found as a function of b/L; as expected, it approaches the Euler stress σ_e for $b/L < 0.1$, and close to $\sigma_e/(1-\mu^2)$ for $b/L > 10$ but, interestingly, never quite reaches the latter value even for infinite b/L. The case when sides b are fixed is studied approximately, with similar results.

L. H. Donnell, USA

1534. Müller-Magyari, F., Contributions to the tension-field-theory of thin-walled plate strips (in German), *Öst. Ing.-Arch.* **4**, 1, 12–27, Feb. 1950.

Buckling and post-buckling behavior of a thin rectangular plate under shear, longitudinal, and cross compressive stresses is analyzed in part I. With the aid of some simplifying assumptions, author is able to determine the stresses for transition from the cylindrical buckling form to the more usual diagonal wave pattern up to loads 25 times the Euler critical. Part II deals with shear buckling of an orthotropic long thin rectangular plate of any orientation of grain. D. C. Drucker, USA

1535. Boley, Bruno A., The shearing rigidity of buckled sheet panels, *J. aero. Sci.* **17**, 6, 356–362, 374, June 1950.

Based on von Kármán's effective width concept, formulas for the effective and reduced effective shear modulus of flat and curved buckled sheet panels are presented. The shearing rigidity of a buckled panel is assumed to be a function of amount of waviness and shape of the waves and that the reduction in shear modulus is due to increased movability afforded by the waves and folds rather than the inelastic behavior of the panel.

By considering that only the effective width strips have load-carrying capacity, theoretical values obtained for effective shear

modulus are somewhat smaller than those obtained from experiment. This reduction is partly attributed to neglect of the middle portion of the panel. Thus to obtain agreement between theory and experiment, the expressions for shear modulus have been modified to include an empirical exponent.

Theoretical results are given for panels subjected to compression, shear, and to combined shear and compression. Comparisons of values of effective shear modulus to experiment for compressive and shear loadings are shown. Theoretical curves for combined shear and compression are also given. Design values for the empirical exponent are suggested.

Elio D'Appolonia, USA

1536. **Winter, George, Performance of thin steel compression flanges,** *Cornell Univ. Engng. Exp. Sta.*, reprint **33**, 27–37, Nov. 1950.

See AMR **2**, Rev. 723.

Joints and Joining Methods

(See also Revs. 1517, 1594, 1600)

1537. **Roberts, Irving, Gaskets and bolted joints,** *J. appl. Mech.* **17**, 2, 169–179, June 1950.

Important innovations in engineering theory and practice often occur in old and well-established fields where the prospects for significant advances may appear slight. Bolted joints for pipes and containers under internal pressure represent one of these groups of ancient elements of machinery for which rational informaton on fundamental behavior has only recently been developed. Present paper is a worth-while contribution to this field. Process of initiation of leakage is examined critically, and for the first time a clear basis is established for the relation between the gasket pressure and internal pressure. The critical examination of a considerable amount of previously published data throws much light on the problem. Paper also contains a considerable amount of rational discussion of elastic deformations in flanges and gaskets. It concludes with a clearly formulated proposal for a design procedure.

C. Richard Soderberg, USA

1538. **Mikhalapov, G. S., Direct explosion of welded joints,** *Weld. Res. Suppl.* **15**, 3, 109–122, Mar. 1950.

In explosion tests, explosion loads of different strength are exploded on plate-shaped bodies, supported only on the edges, and the resulting buckling or fracture observed. The investigations should determine if this testing method is apt to characterize material behavior under multiaxial stresses, especially in welded connections. Tests were carried out on square plates of various steels of 6-ft length and 3-in. thickness at various temperatures, and led to following conclusions: (a) A good correspondence seems to exist, in structures of high stress gradient, between the loading capacity of welded connections and the static fracture load. (b) There seems to be big differences in the work necessary for fracture, and in the plate deformation up to fracture in ferritic steel plates, especially at temperatures below 0 F, dependent on chemical composition and heat treatment. (c) Strength of welded connections is in nearly all cases lower than strength of the plates, drop becoming very great in the cold. (d) Strength of a welded connection made with electrode 6010 from mangansteel is only half of strength of plate at 30 F, and 1/10 at − 40 F. (e) Strength of welded connections can be largely improved through selection of suitable electrodes. (f) Strength of a welded connection cannot be predicted from the properties of plate and admixed material, since it is a complicated function of both.

E. Siebel, Germany

Structures

(See also Revs. 1527, 1528, 1579, 1592, 1610, 1769, 1771, 1829)

1539. **Beaufoy, L. A., and Diwan, A. F. S., Equivalent elastic systems in the analysis of continuous structures, I, II,** *Concr. Constr. Engng.* **65**, 11, 12; 383–389, 427–434; Nov., Dec. 1950.

Describes a method of reducing the whole or part of a structure to an elastically equivalent single fixed-ended system; subsequently method is applied to a numerical example of a continuous frame on flexible columns. Elastic constants are expressed in terms of displacement coefficients which are then related to stiffness factors. Elastic constants for the general case of a continuous frame are discussed and it is shown how to replace an actual structure by a virtually equivalent system. Method is ingenious, but reviewer does not believe it superior to moment distribution since a numerical error in an early part of the analysis affects all subsequent work.

F. L. Singer, USA

1540. **Niles, A. S., Clerk Maxwell and the theory of indeterminate structures,** *Engineering* **170**, 4414, 194–198, Sept. 1950.

Author gives history of development of Maxwell's principle of reciprocal deflections. He gives a modified version of Maxwell's original paper showing the earliest exposition of this important principle, and points out contributions by later writers on this subject, bringing it to the present form as one of the most important principles in structural analysis.

T. H. Lin, USA

1541. **Falkenheiner, H., Systematic calculation of elastic characteristics of statically indeterminate systems** (in French), *Rech. aéro.*, no. 17, 17–31, Sept.-Oct. 1950.

Paper discusses solution by matrix calculus of simultaneous linear equations for highly statically indeterminate systems, with some reference to problems of this type in aircraft design. Hooke's law and linear relation between loads and deformations are assumed. Redundants are introduced in the form of generalized coordinates to obtain symmetrical and positive matrixes of the static influence coefficients, for use in analyzing deformations, vibrations, and "resistance." Author's term "resistance" refers, however, merely to a determination of the stress or internal force distribution on basis of the above linear assumptions. Basic equations are derived in usual manner from reciprocal theorem and principle of least work. Solution of such sets of equations by matrix analysis is by no means new [see, e.g., Beyer, Kurt, "Die Statik im Eisenbetonbau," Stuttgart, 1927], and the special approaches introduced by author require thorough familiarity with the more intricate methods of matrix calculus. It is questionable whether the practical use of this approach is justified as compared to solution by relaxation methods, but it may be useful for setting up the matrixes in a form most convenient for solution by computing machines.

George Winter, USA

1542. **Di Berardino, Vincenzo, and Frandi, Paolo, Recurrence formulas for step-by-step solution of systems of linear algebraic equations** (in Italian), *Ric. sci.* **20**, 5, 662–666, May 1950.

Paper gives chain method of solving systems of linear equations avoiding cumbersome Cramer's arithmetic. It requires same number of operations as in known Banachiewicz method. Its advantage is the use of a chain of elementary algebraic recurrent formulas, simplifying computations, and allowing a better control of partial results. This method was used by Di Berardino for the gradual introduction of unknowns into structures of several

hyperstatics, and by F. Iossa in studying statically indeterminate elastic systems. Maria Castellani, USA

1543. Raymondi, Carlo, Contribution to the study of statically indeterminate elastic systems (in Italian), *Atti Ist. Sci. Contr. Univ. Pisa*, no. 13, 15 pp., 1949.

A statically indeterminate system is reduced to statically indeterminate fundamental systems with the indeterminancy diminishing step by step. Thus, recurrence formulas may be set up for the calculation of the unknowns $x_i^{(n)} = x_i^{(n-1)} - c_{in} \cdot x_n^{(n)}$ starting from the simple statically indeterminate system; the last unknown $x_n^{(n)}$ of each calculation step is obtained from one equation. Two numerical examples are given (there is a mistake in fig. 1c). The same method of solution of linear systems of equations has been recently proposed by V. Di Berardino and P. Frandi (see preceding review), referring to an earlier paper by V. Di Berardino [*Ric. Ingegneria*, 1935]. Method of calculation is clear and simple, but, in reviewer's opinion, it does not save time in comparison with the Gauss solution method.

F. Stüssi, Switzerland

1544. Mudrak, W., Calculation of a star symmetrical one story frame for wind pressure by the deformation method (in German), *Öst. Ing.-Arch.* **3**, 3, 203–215, 1949.

Analysis of statically highly indeterminate systems is substantially simplified by introduction of independent "deformation groups," since the system of equations will then disintegrate to such an extent that at most two simultaneous equations appear. It is possible to determine coefficients for numerical calculations such that after obtaining the dimensions of the system, the final equations can be written down immediately. A numerical example is given. From author's summary

1545. Smith, Frederick, C., and Vincent, George S., Aerodynamic stability of suspension bridges. Part II. Mathematical analysis, *Univ. Wash. Engng. Exp. Sta. Bull.* 116, 62 pp., Oct. 1950.

Complete report on theoretical and experimental studies of the aerodynamic stability of suspension bridges, undertaken at University of Washington, will be divided into five parts. The present one covers only the theoretical analysis of the natural modes of vibration of the bridge and the models, as influenced by their various physical properties.

After a general introduction devoted mainly to a qualitative analysis of the forms of motion and the nomenclature, the free vibrations of the original Tacoma bridge are studied in chapter 1, under following simplifying assumptions: (1) Additional cable tension produced by deflection is small compared with tension due to dead load; (2) extension of cable is negligible; (3) suspenders are inextensible; (4) towers are perfectly flexible to horizontal forces applied at the tops; (5) bending stiffness of suspended structure is neglected. Vibrations studied in chapter 1 are classified into: vertical symmetrical, or asymmetrical bending modes and torsional modes. Effect on corresponding frequencies of unloading the side spans of the bridge are investigated in chapter 2.

The results so obtained are successively improved by replacing the simplifying assumptions (4) and (5) by the actual conditions prevailing in the bridge. Chapter 4 is devoted to the effect of a substantial stiffening truss. In chapter 5, this stiffening truss is assumed to also have substantial torsional rigidity. Chapter 6 investigates the effect of the torsional rigidity of the towers.

A knowledge of the energy of vibration of a given mode is essential if any study of damping and exciting forces is to be made. The corresponding calculations are given in chapter 7 for all modes. Paper concludes with five appendixes, giving some detailed mathematical derivations, numerical examples, and explicit calculation of the torsional rigidity of a suspension-bridge tower. Ch. Massonnet, Belgium

1546. Selberg, A., Calculation of small suspension bridges (in Danish), *Bygnstat. Medd.*, no. 2, 43–62, 1949.

A brief account of the theory and calculation of suspension bridges is given and the use of influence line diagrams for this calculation is shown. The influence line diagrams for increase in cable force, hanger force, and bending moment in the stiffening girder at the middle of span are given. Accuracy of the method is considered and found to be good. Corrections for longitudinal movements of the cables and the effect of inclination of the hangers are discussed and a simple but accurate correction for cable movement is introduced. Effect of hanger inclination may be managed in a simple manner by a correction of the cable force. A short representation of the wind movements of suspension bridges, vertical and torsional oscillations is given with formulas for calculation of the frequencies of these oscillations. Methods for decreasing these oscillations, especially model investigations and stay cables, are discussed.

From author's summary

1547. Guyon, Y., Calculation of plate bridges (in French), *Ann. Ponts Chauss.* **119**, 5, 6; 555–589, 683–718; Sept.-Oct., Nov.-Dec. 1949.

1548. Pei, Minglung, Prismatic structures, *Concr. Const. Engng.* **45**, 7, 235–242, July 1950.

This is one of a series of articles begun in October, 1948, by A. J. Ashdown, on prismatic thin slab structures [AMR **3**, Rev. 463]. Such construction may be used instead of shell-type structures, or for structures such as storage bins. From certain simplifying assumptions, an analysis has been developed using simple beam theory.

Here author criticizes two of the assumptions made by Ashdown. He shows that the distribution of shearing stress along the joint of two slabs is parabolic and not uniform, as assumed by Ashdown. He also shows that Ashdown's assumption for multispan structures leads to inaccurate estimate of stresses at certain points, due to neglect of deformation compatibility at joints. In reply, Ashdown shows that initial simplified approach can, by slight alteration, give sufficiently accurate results.

To reviewer, it appears that this latter modification is only applicable in simple symmetrical cases, such as example treated, and that otherwise author's criticism is valid.

F. A. Blakey, Australia

1549. Leitner, H., The design of multiple-story earthquake resistant buildings, *Concr. Constr. Engng.* **65**, 9, 329–332, Sept. 1950.

1550. Bisplinghoff, R. L., Isakson, G., and Pian, T. H. H., Methods in transient stress analysis, *J. aero. Sci.* **17**, 5, 259–270, May 1950.

Methods of computing the dynamic response of an airplane structure to transient forces are considered. A thorough discussion of various types of generalized coordinates used in dynamic analysis is followed by the development of generalized equations of motion, and author indicates how the equations can be expressed in terms of the normal coordinates. Solution of the equations is then treated for case where the external forces are pre-assigned and for case where the forces depend in part on motion of the system. It is assumed, however, that the external

forces do not introduce nonlinearity in the system. A convenient approximate method for finding transient response from the mechanical admittance function is presented. An important feature of this method is that it is applicable to systems having damping. Several ways of computing stresses from the response in generalized coordinates are presented and their relative merits are discussed. Those readers who have long felt the need for a clear statement of assumptions involved in the mathematical statement of the problem of dynamic response of airplanes, and a systematic introduction to mathematical concepts and methods used to obtain a solution, will find much of value in this paper.

S. Levy, USA

1551. **Pian, T. H. H., and Flomenhoft, H. I., Analytical and experimental studies on dynamic loads in airplane structures during landing,** *J. aero. Sci.* **17,** 12, 765–774, 786, Dec. 1950.

Paper investigates following assumptions commonly made in analysis of transient landing loads: (1) Applied force independent of structure response; (2) structural damping and aerodynamic forces negligible. First assumption is justified by analysis of simplified system. It is shown that in most practical cases underestimate of maximum load does not exceed 10%. Second assumption is checked by drop-testing a flexible model in still air and in wind tunnel. Damping and aerodynamic effects are shown to be small. Experimental results are compared with analysis in which total stress is found by adding vibratory stress to rigid body stress and use is made of mechanical transient analyzer.

Gabriel Isakson, USA

1552. **Epstein, Albert, Nonlinear effects of structural deformation on stability,** *J. aero. Sci.* **18,** 1, 50–54, Jan. 1951.

Paper analyzes certain peculiarities in "stick force per g" curves obtained in flight tests on the Republic F-84, a plane of the fighter class. At an indicated speed of 550 mph, the fitting of empty wing-tip tanks reduced the load factor for reversal of slope of the "stick force per g" curves from 7.0 to 4.5. With full tanks, reversal occurred at 3.5, but the addition of a small stabilizing fin to the tanks brought the reversal point back to 7.0. Author attributes these effects to influence of skin buckling caused by aerodynamic and inertia forces acting on the tanks and applied to the wings. Skin buckling will reduce the "tangent" torsional stiffness of the wing, and by inducing flow separation will, according to data given, produce a nose-up pitching moment. On both these counts skin buckling will reduce stability. These and other effects considered by author are worthy of serious attention by aircraft designers. However, this field of knowledge is one in which much debate is possible, and reviewer would, therefore, like to endorse author's plea for "additional research to fill in some of the obvious gaps."

W. S. Hemp, England

1553. **Neal, B. G., Plastic collapse and shakedown theorems for structures of strain-hardening materials,** *J. aero. Sci.* **17,** 5, 297–306, May 1950.

See AMR **3,** Rev. 2280.

1554. **Symonds, P. S., The basic theorems in the plastic theory of structures,** *J. aero. Sci.* **17,** 10, 669–670, Oct. 1950.

Reader's forum note concerning Neal's paper on plastic collapse and shake-down theorems [AMR **3,** Rev. 2280]. Systems of inequalities for the necessary and sufficient conditions for shake down are presented, simpler than those given by Neal. Physical interpretation of these inequalities is discussed, relating them to two distinct types of continuing plastic flow.

F. J. Plantema, Holland

1555. **Ferguson, Phil M., Analysis of three-dimensional beam-and-girder framing,** *J. Amer. Concr. Inst.* **22,** no. 1, 61–72, Sept. 1950.

Author presents elementary analysis of a system of parallel continuous beams, some supported by transverse girders framed into columns and others by the columns directly, under live loads so disposed as to create either maximum positive or maximum negative moments in the spans considered. The torsional stiffness GJ/L of the girder, which operates similarly to the bending stiffness EI/L of the beam or column, is found to exert a greater influence on the moments than it is usually credited with. Author states that positive live-load moments are apt to be 15 to 35% and negative live-load moments 10 to 15% larger in beams framing into girders than those framing into columns. Reviewer believes there is actually less contrast, due to restraint against rotation of girder exercised by the connecting monolithic slab normally present. Author also gives a tabular form for moment distribution in three-dimensional frames.

A. R. C. Markl, USA

1556. **Neal, Bernard George, and Symonds, Paul Southworth The calculation of collapse loads for framed structures,** *J. Instn. civ. Engrs.*, no. 1, 21–40, Nov. 1950.

Paper presents a new method for determination of collapse load for structures of any degree of complexity by a simple routine analysis. Method is limited only by the time which would be taken in the analysis for a large frame. It is shown, however, that a skilled analyst may be able to reduce the labor involved to a considerable extent. Method is contrasted with method which is used at present, in which designer guesses the locations of plastic hinges which form when the structure collapses, and then adjusts his guesses in a systematic manner.

From authors' summary

1557. **Symonds, Paul Southworth, and Neal, Bernard George, The calculation of failure loads on plane frames under arbitrary loading programmes,** *J. Instn. civ. Engrs.*, no. 1, 41–61, Nov. 1950.

A two-bay rigid-joint bent subjected to a system of vertical and side-sway loads is considered. Loads are not of fixed magnitude but are allowed to vary independently of one another. A shake-down analysis (i.e., a determination of the conditions wherein plastic flow will ultimately cease and be followed by elastic behavior) is effected by supplementing the equations of static equilibrium with certain truths based on bending moment—curvature relationships in the plastic as well as elastic range. In conclusion, a comparison of load parameters for initial yielding, for shake-down, and for plastic collapse indicates that a considerable savings in material would be possible by designing on appropriate plastic theory.

Duane R. Keller, USA

1558. **Prager, W., and Symonds, P. S., Stress analysis in elastic-plastic structures,** *Proc. Symp. appl. Math.* **3,** 187–197, 1950.

General principles of stress analysis of indeterminate elastic-plastic trusses are discussed. Starting from the application of basic concepts to plane trusses, authors describe the orthogonality relations, stressing importance of minimum principles which govern the stress analysis in elastic-plastic trusses, and expressing these concepts in the geometrical language of stress space. The work of G. Colonnetti and H. J. Greenberg is also discussed and their relative merits and defects brought out. Authors conclude with a note on limit design and safety factor for this type of structure. Practical applications of this concept have been very recently dealt with by Neal and Symonds (see preceding two reviews) and should be read along with this paper.

S. K. Ghaswala, India

1559. Goto, Yukimasa, Economical design of a reinforced slab with straight haunches (in Japanese), *J. Soc. civ. Engrs. Japan* **35**, 3, 1–6, Mar. 1950.

Following the usually adopted method for design of reinforced structures, author derives the relations required for the design of both-ends-fixed reinforced-concrete slabs with straight haunches, when the following items are given: (1) Span, (2) equivalent uniform load, and (3) the allowable stresses of the concrete and steel, thermal effects and shrinkage due to drying being neglected. In addition, he studies the conditions to be satisfied if the cross-sectional area is a minimum, which is thought to be most economical. He gives a diagram by which the most economical sections of such a slab can be determined easily.

Takeo Mogami, Japan

1560. Nielsen, Knud E. C., Investigation of load distribution between reinforced concrete floor slabs and their formwork, *Sven. Forskninginst. Cement Betong. Medd.*, no. 19, 15 pp., 1949.

Object of investigation is to study the loads to which concrete floor slabs and their formwork in ordinary dwelling houses are subjected during period of construction. Deflections of floor slabs were recorded both during period of construction and after completion of house. Rate of construction and quality of concrete are taken into account, and a comparison is made between structural actions of several types of floor slabs. A study is made of the influence exerted by swelling and shrinkage of form timber caused by variations in moisture content. From author's summary

1561. Eichstaedt, H. J., Iteration method for stress determination in eccentrically loaded reinforced-concrete cross sections (in German), *Bauplan. Bautech.* **4**, 3, 87–91, Mar. 1950.

Author gives iteration method for determination of position of neutral line. Rapid convergence is illustrated by some examples in which estimated initial position of neutral line is close to actual position. Method seems equivalent to Newton's method for approximate solution of algebraic equation.

W. T. Koiter, Holland

1562. Dehan, E., and Louis, H., Measurement of stresses and their variation in accessible wires in structures of prestressed concrete (in French), *Ann. Trav. publics Belg.* **103**, 2, 201–256, Apr. 1950.

Authors describe practical methods for measuring effect of relaxation, shrinkage, plastic flow on final prestress force. Measurements are made on groups of wires by putting a calibrated pipe between sandwich plates and concrete and reading deformations with dial gages or ohmic strain gages, and on individual wires either by natural frequency methods or by adding a transverse force of known magnitude and measuring the change of direction of a given section. Field tests show that present method of stressing is more of a cause of error than any shrinkage and relaxation. Practical remedial solutions are given.

R. Quintal, Canada

1563. Gough, V. E., Tyre and vehicle behaviour, *Auto. Engr.* **39**, 512, 513; 97–106, 139–145; Mar., Apr. 1949.

1564. Vawter, Jamison, and Clark, James G., Elementary theory and design of flexural members, New York: John Wiley & Sons, Inc.; London: Chapman & Hall, Ltd., 1950, viii + 215 pp. $4.

This textbook is primarily useful for noncivil-engineering students desiring a broad course in structural theory, with some application to design. Beams and columns in both concrete and steel are covered, with numerous illustrative examples. Considerable stress is laid on solution of problems using fundamental principles rather than memorized formulas.

R. L. Janes, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 1494, 1521, 1553, 1554, 1557, 1581, 1614, 1626)

1565. Coffin, L. F., Jr., The flow and fracture of a brittle material, *J. appl. Mech.* **17**, 3, 233–248, Sept. 1950.

Criteria usually applied to plastic flow of ductile metals (constant distortion energy or shear stress) do not hold for brittle materials. Purpose of present paper is to find proper relations for these materials. Experiments are made with tubes of gray cast iron, the brittle material of greatest technical significance, covering the range of plane-combined stress states from tension-tension to compression-compression. The main results are: There exists no unique generalized effective stress-strain curve in the usual sense; the material is quite dissimilar in tension and compression. Volume increases generally during plastic flow, Poisson's ratio is considerably less than $^1/_2$ for tension and greater than $^1/_2$ for compression. Results are interpreted in following manner: Structure of gray cast iron, consisting of graphite flakes in a pearlitic matrix, may be considered as that of a ductile one with randomly dispersed fine cracks. Upon application of an external stress, stress concentrations arise at the sharp ends of the cracks, and it is assumed that flow is determined by a notch effective stress $\sigma_N{}^*$ based on maximum shearing stress occurring in the metal at a crack edge. Although the stress concentration factor may have a high value initially, its value is only about 3 when plastic flow begins. Furthermore, the flakes are under residual compressive stresses because of the different thermal expansion coefficients of graphite and iron, and therefore the matrix is under a residual tensile stress at the ends of a crack which must be added to the notch-raised external stress. The mathematical formation of these assumptions, using the known general relations, indicate nearly a unique generalized stress-strain curve $\sigma_N{}^*$ vs. effective strain ϵ^*, which is closely verified by the experiments except for the compression-compression region at large strains which may be understood as a reduction of the stress concentration factor according to compacting of the flakes normal to the compression stress. A unique stress-strain curve indicates a generalized Hooke's law for plastic deformation in which strains are linear functions of stresses with two Poisson ratios for tensile and compression stresses, respectively. The laws of fracture also can be derived on the basis of these relations. In the tension-tension and the tension-compression regions the fracture takes place at a critical value of $\sigma_N{}^*$ determined by a critical value of the distortion energy in the compression-compression region, and, on the other hand, at a critical shearing stress along the octahedral plane which depends both on the octahedral shearing stress and a frictional shearing stress proportional to the normal stress on the octahedral plane. Finally, the tension characteristics after a preceding compression (increase of the stress and plastic strain at fracture, change of Young's modulus) may be explained with these conceptions, which according to reviewer's opinion seem to give a remarkable contribution to the theory and engineering application of the plastic behavior of brittle materials with heterogeneous inclusions, as in the case of gray cast iron. Albert Kochendörfer, Germany

1566. Hodge, P. G., Jr., Approximate solutions of problems of plane plastic flow, *J. appl. Mech.* **17**, no. 3, 257–264, Sept. 1950.

Stress and strain distributions are determined for metal-forming problems in plane strain. The approximate stress fields consist of regions of uniform stress separated by stress discontinuities

which satisfy equilibrium conditions. These regions move as rigid bodies with slip across the stress discontinuities. Wedge indentation and pressing a slab are analyzed, the former giving results in close approximation with known complete solutions. Deformation fields are simply obtained from properties of affine transformations.

Reviewer believes that value of the paper would be enhanced and use of method promoted by specific statement that approximation lies in slip at stress discontinuities, whereas Mises flow law permits slip only along slip lines, and so approximation is close when lines of stress discontinuity and slip lines are inclined at a small angle.

E. H. Lee, USA

1567. **Coburn, N., A graphical method for solving problems in plane plasticity,** *Proc. Symp. appl. Math.* **3,** 201–211, 1950.

Let $\mathbf{r} = \mathbf{i}x + \mathbf{j}y$ be position vector, $\mathbf{w} = \mathbf{i}u + \mathbf{j}v$ be gradient of Airy stress function. To arbitrary $d\mathbf{r}$, from fixed point P, corresponds $d\mathbf{w}$, with $du = \sigma_y\, dx - \tau_{xy}dy$, $dv = -\tau_{xy}\, dx + \sigma_x dy$, where σ_x, etc. are stresses at P. Hence if $d\mathbf{r}_1$, $d\mathbf{r}_2$ are along principal axes of stress, corresponding to principal stresses σ_1, σ_2, respectively, then $d\mathbf{w}_1 = \sigma_2 d\mathbf{r}_1$; $d\mathbf{w}_2 = \sigma_1 d\mathbf{r}_2$ (A). Author introduces curves β = const, α = const following directions $d\mathbf{r}_1$, $d\mathbf{r}_2$, respectively; approximates differentials in (A) by differences. Terminal points P_1, P_2 are supposed known; initial point $P = P_3$ and corresponding $\mathbf{w}$ and stresses are to be found. (A) gives 4 scalar equations in 6 unknowns, x_3, y_3, u_3, v_3, σ_1, σ_2. Fifth equation $d\mathbf{r}_1 \cdot d\mathbf{r}_2 = 0$, places P_3 on circle with P_1P_2 as diameter. Sixth is plasticity condition, taken as $\sigma_1 - \sigma_2 = k$; it and (A) place P_3 on straight line parallel to, and at known distance from P_1P_2. Continuity determines choice between two intersections. Successive steps, starting from boundary under known tractions, determine points of orthogonal (α, β) net and corresponding stresses. Author carries out solution for internal circular boundary under simultaneous uniform normal and tangential tractions; has not fully tested method in less symmetric cases. Readers dismayed by author's tensor notation may derive (A) by elementary methods and start at equation (25).

William Fuller Brown, Jr., USA

1568. **Hodge, P. G., Jr., and White, G. N., Jr., A quantitative comparison of flow and deformation theories of plasticity,** *J. appl. Mech.* 17, 2, 180–184, June 1950.

Stresses and displacements in a partly plastic tube expanded by internal pressure are calculated under conditions of plane strain. The von Mises yield criterion is used and compressibility is allowed for. Solutions based on the Reuss and Hencky relations are found not to differ greatly. It is concluded also that, in this particular problem, the solutions based on the von Mises and Tresca yield conditions differ negligibly. R. Hill, England

1569. **Nadai, A. L., The flow of metals under various stress conditions** (in German), *Öst. Ing.-Arch.* **3,** 3, 5, 1949.

See AMR **3,** Rev. 59.

1570. **Tapsell, H. J., Forrest, P. G., and Tremain, G. R., Creep due to fluctuating stresses at elevated temperatures,** *Engineering* 170, 4413, 189–190, Aug. 1950.

Authors report results of creep tests on 0.26% carbon steel at 400 C subjected to combined static and alternating stress. For static loading only, geometrically similar strain-time curves up to 200 hr were obtained at various stresses, creep strain at any time being approximately proportional to fifth power of stress. Tests covered ratios of alternating to static stress components from 0 to 3 and mean stresses from 3 to 11 tons/sq in. Results are compared with theoretical values obtained by assuming instantaneous creep rate $C = K\,(M + A \sin \omega t)^n$ where M is static stress and A is alternating stress component. If range of creep from 2 to 200 hr is used, fair correspondence between average test and theoretical creep strains is obtained, which tends to confirm assumption made in the theory. Individual test values of creep strain varied from 60% to 150% of theoretical, average being around 100%. Test results show higher creep strains than theoretical if range from 0 to 100 hr is used as basis; authors suggest this is due to experimental difficulty in maintaining constant mean stress during initial application of alternating load.

A. M. Wahl, USA

1571. **Barkas, Wilfred W., The rheology of fibre assemblages,** "Some rec. develop. Rheol.," London, United Trade Press, 63–89, July 1950.

Author discusses mechanical properties of ordered fiber assemblages, such as wood, in terms of the elastic properties of the constituent fibers. Information regarding structure of the individual hollow fibers obtainable from experiments of swelling in water and contraction on drying is then considered. Author concludes that since it is unlikely that bulk modulus of fiber walls is changed by moisture content while humidity produces large changes in elastic properties of wood, a hydrostatic pressure applied to material must result in large local internal shear strains. Microscopic voids in material also result in volume plasticity when they collapse. Conditions under which a hollow fibril filled with water will collapse on drying are next considered. Lastly, relation between the chemical structure of constituent long-chain molecules and the elastic properties of the fiber is discussed. Author suggests dynamic measurements of elastic properties of these materials may help to explain their mechanical behavior.

H. Kolsky, England

1572. **Finch, L. G., and Greenough, G. B., Residual strains in plastically deformed mild steels,** *Nature* 166, 4221, 508–510, Sept. 1950.

Consists of two separate communications, one from L. G. Finch and the other from G. B. Greenough. Finch measured residual lattice strains in plastically deformed low-carbon steels (0.06–0.15% C, 0.27–0.67 Mn) by means of a back reflection x-ray technique. Tensile specimens having a cross section of 0.1 in. × 0.5 in. were used. A graph showing the change of residual strains for the {211} planes lying parallel to the surface of the specimen is presented. Positive residual strains as high as 0.02% were observed.

Greenough reports results of similar studies on plastically deformed mild steel wire. Conclusion is that his observations as well as those of Finch indicate that, although Heyn intergranular stresses contribute to the observed residual lattice strains, other stress systems must also be present.

W. T. Lankford, Jr., USA

1573. **Hoffman, Charles A., and Yaker, Charles, Effects of an aging treatment on life of small cast vitallium gas-turbine blades,** *Nat. adv. Comm. Aero. tech. Note* 2052, 33 pp., Mar. 1950.

An investigation to determine effects of an aging treatment on life of small cast vitallium gas-turbine blades operated at a blade temperature of approximately 1500 F and a stress of 20,000 lb per sq in. at the blade-failure plane. Twenty blades aged for 48 hr at 1500 F were compared with 33 unaged blades. Aging, which has been reported to harden cast vitallium and to improve stress-rupture life, apparently improved time for initial blade failure, average life, and uniformity of life of blades used in investigation. Lives of the last blades to fail were not apprecia-

bly affected by aging treatment. Statistical analysis of blade-life data did not indicate a significant improvement in mean life or uniformity of life of blades. This fact, however, does not necessarily prove that aging is without beneficial effects, but rather indicates that further investigation is desirable to obtain more conclusive results.

A comparison of lives of unaged vitallium blades with stress-rupture data for cast vitallium bars evaluated at substantially the same conditions indicated a relation between stress-rupture life and blade life. Both samples were progressively hardened by precipitation during operation. After about 35 hr of operation, they were at same hardness, which increased slightly thereafter. Metallurgical examination revealed that blade failure was initiated by intercrystalline cracking. From authors' summary

Failure, Mechanics of Solid State

(See also Revs. 1565, 1569, 1588, 1592, 1593)

1574. Rocca, R., and Bever, M. B., The thermoelastic effect in iron and nickel as a function of temperature, *J. Metals* **188, 2,** 327–333, Feb. 1950.

1575. Hempel, M., and Sander, H. R., Strength properties under tension stress with various load speeds (in German), *Glasers Ann.* **73,** 8, 133–136, Aug. 1949.

1576. Uzhik, G. V., Cohesive strength as a new criterion of strength (in Russian), *Izvestiya,* no. 4, 594–605–628, Apr. 1950.

Papers by author [AMR **2,** Revs. 477, 1269; **3,** Rev. 2655] are discussed in great detail by 13 speakers. Most of them reject the author's papers, since the new theory (a) leads to results which contradict experience, and (b) makes assumptions which are either false or represent a very rough approximation. It is doubted that the author's cohesive strength is a basic property of the material. However, speakers stress the big importance of investigating the cohesive strength. Some of the speakers reproach the author for the purely formal treatment of the subject.

Author replies in a paper. First, the concept of cohesive strength is discussed in more detail, with reference to the previous papers. It is then shown experimentally that the first crack takes place not on the surface of the notch but deeper within the material, where, according to author's assumption, the elastically and plastically deformed zones meet. It has been retorted that the cohesive strength, according to author's definition, must depend upon the elastic limit and upon the definition of this limit in steps ($\sigma_{0\cdot002}$; $\sigma_{0\cdot02}$; $\sigma_{0\cdot2}$). Author defends the viewpoint that the yield point with a permanent elongation of 0.2% represents a natural flow limit. He shows that although a larger change of the yield point strongly influences the exterior stress, at which the break occurs, the cohesive strength, calculated according to his method, changes little. G. Masing, Germany

1577. Ōkubo, H., On the endurance limit of a round bar with longitudinal grooves, *J. appl. Phys.* **21,** 11, 1105–1108, Nov. 1950.

It is known that the endurance limit of a metal does not depend only on the calculated stress when the region of high stress is small. An attempt has been made to modify the theory, in which stress is calculated at a point, so that an average stress is determined over a small interval. The lineal distance considered is assumed to depend mostly on grain size. With this modified theory it is expected that the endurance limit of a metal would be independent of its form and size. If this expectation is assumed true and the endurance limit is known for plain and notched specimens, then it is possible to calculate this distance—which should be nearly constant for specimens taken from a common source. A series of experiments were performed to determine endurance limits of specimens having different shaped stress raisers. Characteristic length of this small interval, obtained from these experiments for mild steel specimens, is 0.21 mm. This compares with 0.23 mm obtained in previously reported experiments. It is concluded that value of this length depends only on the kind and history of the metal and is independent of nature of the experiment. This indicates that if average stress is calculated over a region, the size of which is characteristic of the material, that endurance limit will be determined by this average stress, independent of the size of specimen. Irwin Vigness, USA

1578. van Iterson, F. K., The theoretical fatigue curve (in Dutch), *Ingenieur* **62,** 24, MK 55–59, June 1950.

A formula is given for the known Wöhler curve, which is based upon the assumption that the number of load repetitions is inversely proportional to maximum plastic strain occurring in the event of each cycle. An explanation for this rather arbitrary assumption is omitted. It is stated that formula holds only for mild steels showing a distinct yield point.

J. A. Haringx, Holland

Design Factors, Meaning of Material Tests

(See also Revs. 1438, 1556)

1579. Wierzbicki, W., On the problem of permissible shear stresses in steel structures (in Polish), *Inżyn. Budown.,* 1950.

Purpose of paper is to show that to accept the allowable shear stress k_t as 0.8 of normal working stress is risky and it should be taken as $k_t = 0.5k$.

Author analyzes the problem using as a base his own method for calculating factor of safety, which was published in Polish in 1936 and in French in 1945 ["La securité des constructions considerée comme problème de probabilité," *Ann. Acad. Polon. Sci. tech. Varsovie,* **VII,** 1939–1945]. The relationship $k_t = 0.8k$ is based on the hypothesis of maximum strain, and the relationship $k_t = 0.5k$ on the maximum shear. Author believes hypothesis of the strength of materials should be compared not only from the point of view of mechanics, but also from that of statistics.

The comparative calculations are based on the equation (1) $\Omega' \cdot \Omega_3 = p$, where Ω' indicates the probability that the increase of stress caused by not entirely fulfilled assumptions of the theory of strength of materials, or by divergencies of the constant specific values from mean ones in a given material, will not exceed a certain limit; Ω_3 indicates the probability that the maximum stress in the tensile test specimen will not exceed the ultimate strength R of given material, and p is the index of safety, i.e., the previously assumed probability that breakage will not take place.

The calculation of Ω_3 is based on the graphical diagram of Gauss's law $y = (h/\pi^{1/2})\, e - h^2\Delta R^2$ with $\Delta R = R - R_0$ where R_0 is the mean value and $h = 1{:}m2^{1/2}$ where m is the standard deviation. From the axis of symmetry of the Gauss curve, a segment was measured off OO' equal to R_0, and from O' in the direction of O the stress $\sigma_y = \sigma_0 (1 + \Sigma\alpha_i)$ where σ_0 is the value of stress calculated from the theoretical equation, and α_i the limitary increase of this stress caused by not entirely fulfilled assumptions of the theory of strength of materials, or also by divergencies of the constant specific values, for given material, from their mean value. Hence $\Omega_3 = 0.5 + 0.5\theta\ [h(R_0 - \sigma_y)]$ where θ indicates Laplace's function. If we let ω_i be the probability of increase of

α_i then $\Omega' = \pi\omega_i$ will indicate probability that the stress will not exceed the limitary value of σ_g. For shear stresses we have: $\tau_i = \tau_0(1 + \Sigma\alpha_i)$. Furthermore, there were established the known equations for reduced stresses σ_g', as well for hypothesis of maximum strain as for hypothesis of maximum shear, taking into consideration increase of the stresses in respect to theoretical formulas. From equation (1) there are calculated, for $p = 0.9999$, the stresses σ_0 and τ_0, and the ratio $k_t:k$. The calculations were done for the standard deviation 2%, 4%, and 6% and it was found that for the hypothesis of the maximum strain $k_t = 0.607k$, $k_t = 0.497k$, and $k_t = 0.402k$, respectively, while for the hypothesis of the maximum shear independently from m, it gave a constant value $k_t = 0.5k$. Hence the conclusion is that, for safety, the shear stresses should be not higher than $k_t = 0.5k$, or that basic hypothesis should be hypothesis of maximum shear.

K. Zarankiewicz, Poland

Material Test Techniques

(See also Revs. 1460, 1562, 1587, 1590, 1591, 1608, 1618, 1831, 1868)

1580. **Berthier, R. M., Precise measurement of deformations in compression test** (in French), *Rev. Matér. Constr. (C)*, no. 421, 301-305, Oct. 1950.

A compressometer is described for measuring average compressive strain along the load line of a stocky compression specimen such as a cube of concrete. Compressometer consists of a frame and a dial micrometer to measure displacements relative to frame. Holes are drilled into the loading platens of the testing machine, frame of compressometer is attached to a point on load line just below lower end of specimen, and stem of the dial micrometer is attached to a point just above upper end of specimen. Relative displacement between these two points will be proportional to average compressive strain along the load line except for negligible elastic displacement between points of attachment and nearest end face of specimen. Usefulness of the compressometer is illustrated by compressive stress-strain curves for specimens of concrete and of stone, which show the proportional limit as well as the compressive stress at rupture.

Walter Ramberg, USA

1581. **Luthander, S. and Wallgren, G., Determination of fatigue life with stress cycles of varying amplitude,** *Roy. Swed. Air Board Rep.*, no. 18, transl. no. 10, 20 pp., 1949.

Object is to present a method for predicting life of a structure subjected to repeated load with variable stress limits corresponding to load variations on aircraft. Effects of duration of cycles, frequency, and pauses between loads are neglected on basis of cited tests indicating lack of significance. Analysis is based on hypothesis of Palmgren (1924) and of Langer (1937) that failure will occur when the summation of the ratios of number of cycles at a given stress level to the number of cycles at that level causing failure is equal to unity. This hypothesis neglects the so-called training effect shown to be significant for steel and Alclad.

The stress pattern investigated was the one presented by Kaul (1938) for loads on an airplane flying a straight level course for 100 hr at 350 km/h, and is symmetrical with respect to a mean stress. Tests were made on Alclad sheet specimens, some plane and some containing two holes on the transverse center line. Authors conclude that there is fairly good agreement between calculated and experimental values. Sixteen references are cited.

Glenn Murphy, USA

1582. **Bruce, F. M., A photographic method for displacement time recording,** *Brit. J. appl. Phys.* **1**, 11, 291-293, Nov. 1950.

Small spherical reflectors attached to or formed upon the surface of a moving body produce a point image of a fixed light source. This image moves with body to which reflector is attached, and by recording its motion on a continuous-feed or rotating-drum camera, a displacement-time record is obtained in the form of a fine and sharply focused continuous line which is suitable for detailed analysis. Recording may be carried out under normal lighting conditions by this technique. Camera described gives time scales in range 0 to 315 in./sec.

From author's summary

1583. **Petersen, Cord, Measurement of mechanical damping of metals** (in German), *Arch. tech. Messen*, no. 164, V9115-6, Sept. 1949.

Paper reviews various methods of measuring internal friction. It is prefaced by a general discussion of damping, indicating how it is evidenced, the usual measurement unit (logarithmic decrement), and the cause of damping. Factors which must be considered in its measurement and factors of which it is often a function are mentioned.

Six measuring procedures are reviewed, from static measurement of the damping loop to heat gain methods. Description of the action of specific machines is restricted to six kinds, including the optical loop recording machine of Lehr. Presentation is well supported by an extensive bibliography.

J. M. Robertson, USA

1584. **Michels, A., and Wassenaar, J. Ph., An instrument for the calibration of dynamometers,** *Appl. sci. Res. Sec. A*, **2**, 4, 245-248, 1950.

An instrument is described for the calibration of dynamometers with which forces can be measured between 100 and 30,000 kg with a reproducibility varying from 0.5 to 3 kg over this range. This accuracy is obtained by applying the principle of the rotating piston as developed for pressure balances.

From authors' summary

1585. **Bouman, H. B., Beekhuis, D. A., and Toneman, F. H., An instrument for the calibration of dynamometers,** *Appl. sci. Res. Sec. A*, **2**, 4, 269-271, 1950.

Using instrument described in the preceding article, the calibration of a proving ring has been carried out. Results show that this method of calibration has advantages over the classical procedure of direct loading at fixed intervals.

From authors' summary

Mechanical Properties of Specific Materials

(See also Revs. 1531, 1571, 1577, 1578, 1580, 1581, 1583, 1806)

1586. **Staff, C. E., Quackenbos, H. M., Jr., and Hill, J. M., Long-time tension and creep tests of plastics,** *Trans. Amer. Soc. mech. Engrs.* **72**, 5, 697-704, July 1950.

Paper reports on long-time tension-creep tests on thermosetting and thermoplastic materials. Tests were run at 25 C and 75 C for periods ranging from 1000 to 14,500 hr. The relative humidity was controlled at 25 C, but not at 75 C.

SR-4 electric strain gages were used to measure creep strain and these readings were checked in some cases with dial gage readings. Secondary objective of this study was to provide information on the reliability of an electric gage over a long period of time.

The equation $\log_{10}(\epsilon/\sigma) = 0.103 \log_{10} t + 0.713$ provides a

quantitative relation between creep and stress for four materials having fillers of wood flour, fabric, paper, and floc. It was found that some creep curves are linear when plotted on rectangular coordinates, whereas others are linear when plotted on logarithmic coordinates. Presumably, no single equation could cover all the materials tested. Authors think that tensile creep specimens fracture when a critical tensile breaking strain is reached. A "5-year" allowable stress is calculated on this basis.

This study also provides valuable information on the technique of using wire gages for the study of creep in plastics.

Yoh-Han Pao, USA

1587. Stavrolakis, J. A., and Norton, F. H., Measurement of the torsion properties of alumina and zirconia at elevated temperatures, *J. Amer. ceram. Soc.* **33,** 9, 263–268, Sept. 1950.

A precise instrument for measuring torsion properties of refractories up to temperatures of 1500 C is described. Twist is measured by sapphire mirrors mounted on specimen itself. Mechanical properties of dense alumina and dense stabilized zirconia were measured at various temperatures. Higher ultimate strengths were reached than those reported for other bodies.

From authors' summary

1588. Lessells, J. M., and Jacques, H. E., Effect of fatigue on transition temperature of steel, *Weld. Res. Suppl.* **15,** 2, 74–83, Feb. 1950.

Report concerns an investigation to determine if prior fatigue would affect the transition-temperature curves in impact of ship steels. Tests were made on two shipbuilding steels, B and W. A combination fatigue-impact specimen was designed and used, it being round and with a circumferential notch, and could be tested in impact by sawing off the tapered ends necessary for holding in the fatigue machine.

As testing proceeded, it was found that fatigue cracks were developing at the base of the notch, both above and below the endurance limit. When specimens were cyclically stressed so as to avoid fatigue cracks, the resulting transition curve showed little deviation from the original curve, presumably due to the low stress of the prior fatigue.

A series of specimens of steel B were prestrained in tension prior to testing in impact, with a marked shift of the transition curve resulting.

From authors' summary

1589. Glikman, L. A., Zhuravlev, V. A., and Snezhkova, T. N., Change of damping at cyclical stresses below and above the fatigue limit (in Russian), *Zh. tekh. Fiz.* **19,** 4, 448–464, Apr. 1949.

1590. Roos, P. K., Lemmon, D. C., and Ransom, J. T., Influence of type of machine, range of speed, and specimen shape on fatigue test data, *Amer. Soc. Test. Mat. Bull.*, no. 158, 63–65, May 1949.

Fatigue tests were made on flat and round specimens of an SAE 4340 steel in pure reversed plane bending at 1800 rpm on a Sonntag universal fatigue-testing machine; and on round specimens acting as rotating beams on R. R. Moore fatigue machines at 1800 rpm and 10,000 rpm. Data show that a higher endurance limit is obtained in pure reversed plane bending tests than in rotating-beam tests; in round specimens subjected to pure reversed plane bending than in flat specimens similarly tested; and in rotating-beam tests performed at 10,000 rpm than in those performed at 1800 rpm.

From authors' summary

1591. Oberg, T. T., and Trapp, W. J., High-stress fatigue of alloy steels, *Prod. Engng.* **22,** 1, 159–161, Jan. 1951.

Recent tests were made to determine the fatigue strength of three aircraft steels by means of reversed bending and axial loading. The rate of loading was varied between 90 and 3,450 cpm. The results of these tests are shown in three graphs.

From authors' summary

1592. Lundberg, K. O., and Wallgren, Gunnar G. E., A study of some factors affecting the fatigue life of aircraft parts with application to structural elements of 24S-T and 75S-T aluminum alloys, *Flygtekn. Forsoksanst. Medd.*, FFA no. 30, 33 pp., 1949.

Report is concerned with design of aircraft parts of 24S-T and 75S-T aluminum alloys under fatigue conditions produced by gust loads, maneuvering loads, and pressure cabin loads. Following a survey of procedures of load analysis (load spectra and load factors), results of fatigue tests of plain and riveted strip specimens, representative of different parts of aircraft, are reported in the form of S-log N diagrams. These results are then correlated with load spectra with aid of the additive cumulative damage concept. Conclusion is reached that under fatigue loading, use of the 75S-T alloy will lead to weight savings for plain structural elements only, while even for the moderate stress concentrations unavoidable in simple riveted connections the fatigue life of 24S-T elements will be longer than that of corresponding 75S-T elements, so that no saving in weight can be expected from the use of the stronger alloy.

While the ideas underlying the investigation are modern, the fatigue-testing procedure and the presentation and interpretation of results are disappointingly conventional, disregarding completely the statistical character of fatigue.

A. M. Freudenthal, USA

1593. Dolan, T. J., McClow, J. H., and Craig, W. J., The influence of shape of cross section on the flexural fatigue strength of steel, *Trans. Amer. Soc. mech. Engrs.* **72,** 5, 469–477, July 1950.

A series of tests under carefully controlled conditions are reported in which the influence of shape of cross section on flexural fatigue strength of two types of steel is investigated. A normalized Mayari-R steel and a heat-treated SAE 4340 steel were machined into specimens having circular, square, diamond, and modified diamond (where the extreme fibers were milled off) cross sections. In order of decreasing endurance limits for both steels, the general trend of best results were found to be round, diamond, modified diamond, and square.

Various factors thought to contribute to this effect included (a) influence of relative amounts of cross section at high stress levels from statistical considerations, (b) possible regions of stress concentrations on sections lacking lateral support, (c) varying amounts of residual stresses, (d) variations in degree of cold working induced by different machining operations, and (e) influence of cross-section shape in controlling the relative amount of "localized" inelastic action occurring on the extreme fibers of the beam.

Authors examine thoroughly each of these items with the possible exception of (a), and conclude from their results that the principal controlling factor seems to be due to (e). If a certain proportion of the crystals in the outer fibers are strained inelastically during the test and, following Orowan, the mechanism of fatigue damage is due to a critical amount of localized inelastic stress, shape of cross section can then play an important role. Each type of cross section will have differing degrees of departure of applied bending moment from linearity (assuming constant yield stress), and hence, for the same external flexural strain, fatigue strength (computed from elastic behavior) will depend on cross-section shape. Since circular cross sections have less departure from linearity than square shapes during initial plastic flow (since there is less metal on outer fibers), the computed

fatigue strength is higher. Item (b) is thought to have some influence on this concept, particularly in the case of the diamond-shaped cross section.

Little attention is given to a possible interpretation of these results from a statistical point of view.

Louis F. Coffin, Jr., USA

1594. **Roš, M., Fatigue tests on hollow rods from pure weld-material, "Arcos-Stabilend-B" and "Arcos-Ductilend-55" at multiaxial stresses** (in German), *Schweiz. Arch.* **16**, 7, 193–199, July 1950.

Most welded structures are subjected to fatigue stresses of multiaxial character. Knowledge of the strength of such structures must be based upon comprehensive theoretical and experimental investigations. Author suggests that important information can be uncovered by studying the pure weld material. Two different weld materials are tested in respect to their static strength and ductility and their fatigue strength under multiaxial stresses. Ultimate strength theories applicable to the test facts are discussed. Important conclusions are inferred that increase in fatigue strength must be weighed against loss in ductility.

R. Nilson, Sweden

1595. **Miller, James A., Stress-strain and elongation graphs for Alclad aluminum-alloy 24S-T86 sheet,** *Nat. adv. Comm. Aero. tech. Note* 2094, 31 pp., May 1950.

Results of tests on duplicate longitudinal and transverse specimens of Alclad aluminum-alloy 24S-T86 sheets with nominal thicknesses of 0.032, 0.064, and 0.125 in. are presented in the following form: Tensile and compressive stress-strain graphs and stress-deviation graphs to a strain of about 1%; stress-strain graphs for tensile specimens tested to failure; graphs of local elongation and of elongation against gage length for tensile specimens tested to fracture. The stress-strain and stress-deviation graphs are plotted on a dimensionless basis to make them applicable to sheets of this alloy with yield strengths which differ from those of the test specimens.

From author's summary by Walter Ramberg, USA

1596. **Bogardus, K. O., Stickley, G. W., and Howell, F. M., A review of information on the mechanical properties of aluminum alloys at low temperatures,** *Nat. adv. Comm. Aero. tech. Note* 2082, 65 pp., May 1950.

A thorough survey of available data on commercial aluminum alloys has been summarized in useful form. Properties increased by low temperature are tensile strength, yield strength, modulus of elasticity, hardness, elongation, and fatigue strength. Notch sensitivity in impact is not adversely affected.

F. R. Shanley, USA

1597. **Rosenholtz, Joseph L., and Smith, Dudley T., The effect of compressive stresses on the linear thermal expansion of magnesium and steel,** *J. appl. Phys.* **21**, 5, 396–399, May 1950.

A series of cylinders of annealed, extruded, pure magnesium and of medium soft steel were subjected to a sequence of uniaxial compressive stresses at room temperature while their respective coefficients of linear thermal expansion in the direction of stress application were determined for the range 20 C to 100 C. Specimens were then heat treated to permit either recovery or recrystallization and resulting changes in length were measured. For each material there are given characteristic curves showing the variation of the thermal expansion coefficient and length changes caused by heating as functions of the (true) compressive stress. It is shown that these properties can be used to determine the stress history of a specimen provided the characteristic curves for that material are available. It is suggested that this method of analysis may find application in study of rock deformations and in the investigation of metals, both before and after failure.

From authors' summary by Merit P. White

1598. **McKeown, J., Tensile tests at elevated temperatures on forged D.T.D. 364A and R.R. 59,** *Metallurgia, Manchr.* **42**, 249, 92–96, July 1950.

Author presents results of tensile tests, carried out at elevated temperatures, on forged bars of D.T.D. 364A and R.R. 59, prepared from cast billets, with and without intermediate extrusion.

From author's summary

1599. **Kuntze, W., Dependence of the elastic strain coefficient of copper on the pre-treatment,** *Nat. adv. Comm. Aero. tech. Memo.* 1287, 18 pp., Aug. 1950.

1600. **Mahla, E. M., and Hitchcock, R. B., Effect of welding on the properties of titanium-carbon alloys,** *Weld. Res. Suppl.* **15**, 11, 544–551, Nov. 1950.

Inert-gas shielded tungsten arc process is satisfactory for welding titanium carbon alloys. A carbon content of 0.25% represents maximum for ductile welds in the as-welded condition.

From authors' summary

1601. **Roš, M., The effect of an addition of Frioplast on the structural properties of concrete** (in German), *Eidgenoss. Mat-PrufAnst. Ber., Rep.* 159, 23 pp., Nov. 1948.

1602. **L'Hermite, R., Recent investigations on concrete** (in Spanish), *Inst. Tecn. Constr. Cem.*, no. 94, 40 pp., May 1950.

Paper attempts to determine the principles which lead to a logical and scientific classification of the properties of fresh concrete, and it is shown that the laws and units of mechanics make this possible. This is only a first attempt which requires further perfection. Some experiments are briefly described.

From author's summary

1603. **Sauer, J. A., and Oliphant, W. J., Damping and resonant load-carrying capacities of polystyrene and other high polymers,** *Proc. Amer. Soc. Test. Mat.* 49, 1119–1138, 1949.

Investigation is concerned with experimental determination of the relation between damping capacity and stress for specimens of polystyrene and some other thermoplastic and thermosetting materials. Damping capacities of these materials are determined by a resonant vibration method and are found to increase with stress amplitude approximately to the 2.3 power. Among thermoplastic materials tested, cellulose acetate butyrate is found to have highest damping capacity and polystyrene the lowest. Various grades of phenolic laminates show still lower energy absorptions under dynamic tension-compression loading.

An attempt is made to evaluate the design importance of both damping capacity and fatigue strength by setting up an analysis for determination of the relative load-carrying capacities of a simple member subject to dynamic loads oscillating at resonant frequency. Despite some rather drastic assumptions, comparative rating of the different plastic materials based on resonant load factors would appear to be more significant than comparative ratings based solely on endurance strength or solely on damping capacity. Among the thermoplastics, methyl methacrylate exhibits the highest resonant load values and is about comparable in this respect to the paper-base phenolic laminates. For polystyrene, it is observed that damping capacity varies with stress amplitude in the same manner as experimental creep rate

values vary with applied tensile stress. Comparison of data shows that creep rate increases approximately as square of damping capacity.

From authors' summary

1604. Findley, Willam N., Comments on creep and damping properties of polystyrene, *J. appl. Phys.* 21, 3, 258–260, Mar. 1950.

1605. Weir, C. E., Leser, W. H., and Wood, L. A., Crystallization and second-order transition in silicone rubbers, *J. Res. nat. Bur. Stands.* **44,** 4, 367–372, Apr. 1950.

During an investigation to determine which rubbers might be suitable for use at low temperatures, interferometric measurements of the length-temperature relationships of silicone rubbers have been made. Crystallization was found between −60 and −67 C in Dow-Corning Silastic X-6160 and in General Electric 9979G silicone rubber, the latter of which contains no filler. Crystallization between −75° and −85° was found in Silastic 250. Melting occurred over a range of temperature above the temperature of crystallization. The volume change on crystallization varied from 2.0 to 7.8%. No crystallization or melting phenomena were observed in Silastic X-6073 between −180 and +100 C. All types of silicone rubber exhibited a second-order transition at about −123 C, the lowest temperature at which such a transition has been observed in a polymer. The coefficient of linear thermal expansion of silicone rubbers containing no filler was found to be about 40×10^{-5}/deg C between −35 and 0 C.

From authors' summary

1606. Gailus, W. J., Yurenka, Steven, and Dietz, A. G. H., Strength-variance studies of plastics, *Trans. Amer. Soc. mech. Engrs.* **72,** 3, 299–307, Apr. 1950.

A statistical "variance" analysis is presented for evaluating variables affecting reliability and reproducibility of strength measurement of plastics. Authors present test data of bending strengths (psi) of numerous specimens of polymethyl methacrylate. Variations in results are attributed to such factors as manufacturer, speed of loading, thickness of specimen, and location of specimen in original sheet. Interaction of each factor on one or more of the remaining factors is calculated in relative numerical quantities termed "variance units."

Reviewer believes that this analysis may indicate trends not otherwise apparent, but has the disadvantage that numerical results are purely comparative and have no absolute physical significance.

A. Yorgiadis, USA

1607. Carey, R. H., Stress cracking of polyethylene, *Amer. Soc. Test. Mat. Bull.*, no. 167, 56–58, July 1950.

Effects of some environments upon tensile properties of polyethylene are reported. It is found that tension specimens containing a small hole break at stresses and elongations less than those normally reported. This change in mechanical properties is greatly accentuated by fluids such as alcohols and toluene. Phenomenon is compared to "stress-corrosion cracking" as observed in metals where cracking occurs in mildly corrosive environments. Data are reported for a wide range of molecular weight polyethylene resins, and the effects of environment are found to decrease with increasing molecular weight of the resin. Data permit quantitative comparison of cracking resistance of different resins.

Properties of polyethylene are not of a rubberlike or elastomeric material but more nearly resemble some nonferrous metals. This comparison is inferred from its crystalline structure, stress-strain diagram, and stress-corrosion cracking phenomenon.

From author's summary

1608. Lazan, B. J., Dynamic creep and rupture properties of temperature-resistant materials under tensile fatigue stress, *Proc. Amer. Soc. Test. Mat.* **49,** 757–787, 1949.

Limitations of static testing and importance of dynamic creep and rupture properties in designing for high-temperature service are discussed. Newly developed dynamic testing machines and measuring equipment for determining creep and rupture properties are described. Data on several temperature-resistant materials are presented within mean-stress alternating-stress coordinates to show the influence of dynamic stress on creep and time to rupture. Relationships between testing temperature and dynamic stress influence on creep and rupture are shown. The increased creep and rupture resistance during some of the dynamic tests is discussed in terms of possible metallurgical changes caused by cyclic stress. Data presented show the greatly decreased ductility caused by superposition of cyclic stress on tensile preload.

From author's summary

1609. Duckworth, W. H., Precise tensile properties of ceramic bodies, *J. Amer. ceram. Soc.* **34,** 1, 1–9, Jan. 1951.

In the case of ceramic materials it is difficult to measure tensile strength accurately with the ordinary tensile test because of localized bending stresses due to eccentric loading and untrue specimens. It is implied that some authoritatively reported tensile-strength values for ceramics have been half of their true values. Article makes a strong case for a bending test with two-point loading, instead of a purely tensile test, in finding precise tensile data for brittle materials. For accurate results, these materials must have stresses directly proportional to strain all the way up to the point of rupture, but the two factors of proportionality (moduli of elasticity) in tension and compression need not be equal. In practice it is recommended that electrical strain gages be used to measure strain in top and bottom surfaces of specimen as load is applied. Maximum strains are used in the tensile-strength formula given by author. There is a good discussion of testing of brittle materials and a comparison of results obtained by conventional tensile tests with those obtained by this refined bending test. The investigation and its results take on added importance because of the greatly expanded research program on ceramic-like materials for gas turbines.

W. C. Johnson, Jr., USA

1610. Hognestad, Eivind, and Siess, C. P., Effect of entrained air on bond between concrete and reinforcing steel, *J. Amer. Concr. Inst.* **21,** 8, 649–667, Apr. 1950.

Although air-entrained concrete is being more widely used in structural engineering during recent years to improve workability, laboratory studies of this material have been largely limited to its strength and deformation properties. Little is known about the interaction and bond of this material with steel, as in reinforced concrete work. Authors carry out some pull-out tests on two types and sizes of deformed bars in various positions in a few concrete mixes containing up to 8% of air.

With vertically cast bars the bond strength decreases with greater air content, but the reduction is only about one half that of the corresponding compressive strength and modulus of rupture. For horizontally cast bars a similar decrease in bond strength is recorded beyond about 4% of air. In practically all tests the bond strength of horizontally cast bars is much less than that of verticall cast ones of the same type. This appears to be due to settlement of the wet concrete under the bars, which would be confirmed by the drop in bond strength observed with greater depth of concrete under the bars as had also been found before [Menzel, A. C., *J. Amer. Concr. Inst.* **35**, p. 517, 1939].

Since plain bars are used much more widely as concrete rein-

forcement in most countries than deformed bars, it is unfortunate the authors have not carried out any comparative tests on them.

G. G. Meyerhof, England

Mechanics of Forming and Cutting

(See also Rev. 1566)

1611. **Carrier, G. F., The use of skewed rolls in calendering operations,** *J. appl. Mech.* **17,** 4, 446–447, Dec. 1950.

An angle of skew between cylindrical calendering rolls may be used to partially compensate for thickness variation that results from bending deflection of the rolls. Effect of the angle of skew on thickness variation is analyzed for the conditions: (1) Roll pressure varies inversely as thickness of sheet between rolls, and (2) skew angle is adjusted to provide a sheet thickness variation with the least square deviation from the mean. It is found that for a small ratio of sheet thickness to roll diameter and typical pressure requirement, very effective compensation for roll deflection is possible. In an example, thickness variation was reduced from 20 to 2% with an angle of skew of 0.022 radius.

Reviewer was able to check validity of the results, although a number of errors exist in the text of the intermediate equations.

William Schroeder, USA

1612. **Navarro, J., Contribution to the study of plastic deformation of metals in drawing and wire drawing** (in French), *Rev. Métall.* **47,** 8, 601–613, Aug. 1950.

Paper gives general experimental study of influence of die and drawing characteristics on mechanical properties of resulting wire for conical dies. Die profile and drawing defects associated with it are considered, giving optimum die angle and cylindrical bearing length. Elongation per pass and speed are also considered. Results are given for many metals and alloys.

Large reductions per pass are recommended for more uniform deformation through the section. A limit on speed is suggested. Effect of complete history of drawing procedure on final product is emphasized.

E. H. Lee, USA

1613. **Northcott, L., McLean, D., and Lee, O. R. J., The effect of single- and multi-hole die extrusion on the properties of extruded aluminium alloy bar,** *J. Inst. Metals* **74,** part 2, 81–93, Feb. 1947.

Longitudinal streaks on surface of aluminum-alloy components machined from extruded bar in BA 35 and D.T.D. 423A alloys were found to be associated with erratic circumferential strength. Streaks have been shown to be due to flow structure originating in extrusion through multihole die plates, and the principles will apply to alloys generally. To investigate the problem fully, billets of the two alloys were extruded through die plates having 1, 2, 3, or 4 holes, all the multiholes being symmetrically disposed about the center. Transverse sections of bars extruded through multihole die plates showed a structure termed "radial flow," converging to that portion of surface marked by streak; this structure was absent from bars extruded through single-hole die plates. Segregation of particle constituents was found near periphery in zone of radial flow and, consequently, circumferential tensile properties of multihole material were inferior to those of single-hole bar. Small-scale experiments with lead-bismuth composites and colored plasticine showed that the segregate originated from axial zone of billet; with a die plate having a single central hole, on the other hand, axial segregate was confined to axis of single bar, where it is relatively harmless. Provision of a central hole in 3- and 4-hole die plates confined segregate to the axis of central bar, and this advantage has been confirmed by industrial trials.

From authors' summary

1614. **Bourne, L., Met, B., and Hill, R., On the correlation of the directional properties of rolled sheet in tension and cupping tests,** *Phil. Mag.* **41,** 7th ser., 318, 671–681, July 1950.

Plastic anisotropy in rolled sheet is examined by tension and cupping tests. The earing positions are correlated with the strain-ratios measured in tension tests at various orientations to the direction of rolling. The materials used are copper giving four ears at 45°, brass giving four ears at 50°, and brass giving six ears at 0° and 60°. A theory of plastic anisotropy due to Hill is found to be in good agreement with experimental data for materials producing four ears. The theory is extended to describe more complex states of anisotropy.

From authors' summary

1615. **Cook, Maurice, and Larke, Eustafe C., Calculation of loads involved in metal strip rolling,** *J. Inst. Metals* **74,** part 2, 55–80, Feb. 1947.

Derivation of method is based on assumption that the magnitude of the pure work of rolling is independent of number of passes used in effecting a given reduction in thickness, and the basic experimental data required consist only of a few measured values of rolling loads developed in rolling one material under a series of different sets of rolling conditions. From these data, rolling loads can then be readily computed for practices involving any number of passes, or a sequence of reductions in thickness can be established for which a constant load is developed in each pass.

From authors' summary

1616. **Tolansky, S., Interferometric study of metal surfaces,** *Metal Treatm.* **16,** 60, 195–203, 1949–50.

Author describes principles and techniques of interferometry, all of which have been developed in his own laboratories. Much of the physics of the optical phenomena has been omitted but he illustrates how interferometry may be applied to the study of metal surfaces.

From author's summary

1617. **Armstrong, E. T., Cosler, A. S., Jr., and Katz, E. F., Machining of heated metals,** *Trans. Amer. Soc. mech. Engrs.* **73,** 1, 35–42, Jan. 1951.

Studies are reported of the machinability of several materials at elevated temperatures. It was found that tool life, cutting austenitic stainless steel, was increased tenfold by heating to 400 F. High-temperature alloys, including vitallium, machined freely at temperatures from 700 F to 2000 F. Long curling chips and a smooth cleanly cut surface were produced in hot-machining. The same materials cut at room temperature developed a glazed uneven surface, and the chips were powdery. Austenitic manganese steel machined easily at 1200 F, as did fully hardened high-speed steel. An arc heating method was developed which permitted continuous heating while machining, without heating the work throughout.

From authors' summary

1618. **Henriksen, E. K., Residual stresses in machined surfaces,** *Trans. Amer. Soc. mech. Engrs.* **73,** 1, 69–76, Jan. 1951.

Paper, which reveals that extremely high surface stresses (up to 100,000 psi) can be produced in ductile metals by machining, should prove of particular interest to experimenters concerned with the determination of residual stresses by the known method of planing off layers and measuring the attendant distortion, inasmuch as it indicates a source of possible major error in applying this method. Author reasons that, in view of their magnitude and location near the surface, these stress concentrations also merit consideration by the designer and manufacturer.

In the introductory paragraphs it is demonstrated that these stresses cannot be caused by heating of the chip and tool, but

must be the direct effect of surface grain distortion. The cutting edge does not, as is widely believed, shear off the grains directly; instead, the upper edges of embedded grains are pulled out into thin strings parallel with the surface and bent around the cutting edge before they are broken apart. While the attendant stresses are relieved by elastic action as the tool moves away, this relief is only minor, and stresses approaching true tensile strength of the material are left in a thin surface layer. Detailed test evaluations are given for square-end and side-cutting tools. For the former, it is shown that residual stresses are the lower, the finer the cut, the higher the carbon content, and the greater the back-rake angle. For a side-cutting tool of a specific shape, i.e., given side- and end-cutting angles and nose radius, it is shown that an 18° normal rake angle produces minimum resultant stresses.

A. R. C. Markl, USA

Hydraulics; Cavitation; Transport

(See also Revs. 1638, 1788, 1791, 1869, 1870, 1871)

1619. Marchetti, Mario, The calculation of energy loss in elastically deformable circular conduits (in Italian), *Energia elett.* **27,** 2, 73–76, Feb. 1950.

Conduit is assumed horizontally straight and of circular sections of identical diameter under same internal pressure. Motion is steady and discharge is constant. Simple theoretical solution gives expression of total energy at any point as function of location, of outlet or inlet diameter, and of a coefficient of transverse deformation of sections. Coefficient of frictional resistance results are expressed in terms of energy loss and of the elements of only one terminal section. The difference in discharge resulting from consideration of deformability of conduit as against indeformability may not be negligible, especially when deducing laws of resistance from experimental data.

From author's summary by M. St. Denis, USA

1620. Powell, R. W., Resistance to flow in smooth channels, *Trans. Amer. geophys. Un.* **30,** 6, 875–878, Dec. 1949.

Distinction is drawn between smooth and rough channel flow, and formulas are developed giving Chezy's constant for smooth channels in terms of the Reynolds number, for nonturbulent and ultrarapid flow.

From author's summary

1621. Raytchine, N., and Chatelain, P., Graphic determination of back-water curves, *Houille blanche* **5,** 3, 373–379, May–June 1950.

Authors divide the streams (natural rivers or canals) into sections, the characteristics of which vary little, and apply Bernoulli's equation graphically to the two ends of a section. They thus obtain "characteristic curves" which enable them to determine the peculiarities of the flow. Authors apply their method to two actual cases, the Manosque canal and the Castelman dam.

Translation from authors' summary

1622. Kirk, David B., The effect of transmission distance on the stability of flow-control processes, *Instruments* **23,** 11, 1191–1194, Nov. 1950.

Author investigated, experimentally, stability of flow in a closed-loop flow-control system equipped with orifice plate, differential pressure transmitter, pneumatic controller, and diaphragm motor valve as used in modern refineries and chemical process plants. Universal adaptation of the central control panel makes necessary long pneumatic transmission lines in the control loop which causes stability problems. Author suggests keeping these transmission lines short by placing controller near point of measurement and using a remote means of setting control points from the central control room. Influence of gain, phase displacement characteristics, proportional band, etc., on stability are discussed in practical terms.

O. E. Teichmann, USA

1623. Escande, Leopold, Influence of the duration of the operation on the amplitude of the oscillations in a surge tank (in French), *C. R. Acad. Sci. Paris* **230, 22,** 1932–1934, May 1950.

In some surge-tank installations, duration of the operation is not negligible in comparison to the period of the surge tank, and it is necessary to take it into account especially for the restricted-inlet and multiple-section type of surge tank. Author considers case of surge tanks with restricted inlet and constant cross section; he gives the relative increase of rise (gate closure) or fall (gate opening) of water level as a function of p_0 and θ, when r_0 has the optimum value corresponding to instantaneous gate operation (notation of Calame and Gaden).

A. Craya, USA

1624. Dmitriev, G. T., Calculation of characteristics of a steady smoothly changing motion in open prismatic channels (in Russian), *Dokladi Akad. Nauk SSSR* **68,** 5, 825–827, Oct. 1949.

Author starts from the differential equation of S. A. Christianovich ["Some new problems of a continuous medium," 1938] for a nonsteady motion: $dL = [v + (g\omega/\alpha B)^{1/2}]dt$ (1), where L is length, t time, v velocity in a cross section of surface ω, B is width, α Coriolis number, g gravity acceleration. Signs + or − refer to rising or falling waves. By means of Froude's number $Fr = \alpha v^2/gl = \alpha Q^2 B/g\omega^3$, author writes (1) as $dL = (g\omega/\alpha B)^{1/2}[1 + (Fr)^{1/2}]dt$ (2).

He then considers three cases of slope of the bed $i \gtreqless 0$, and by means of relations of I. I. Agroskin for a steady continuously changing motion [I. I. Agroskin, "Hydraulics," 1944, chap. XVII], for instance, for $i > 0$ $i\,dL = (1 - Fr)dh/(1 - Fr/Fr')$, and for a falling wave he eliminates dL. Here $Fr' = \alpha Q'^2 B/g\omega^3$, where Q' is discharge through given section for steady motion [Agroskin distinguishes the steady, accelerated, and retarded motion by $(Fr'/Fr) \gtreqless 1$].

Then author obtains (for $i > 0$ and falling wave)

$$dt = (a/i)(\alpha B/g\omega)^{1/2}[1 - (Fr')^{1/2}/(1 - z^n) - (Fr')^{1/2}/(1 + z^{n/2})]dz$$

where $z = (Fr'/Fr)^{1/n}$ and $a = dh/dz$.

Integrating between the limits z_1, z_2, we get

$$t = (a/i)(\alpha B/g\omega)^{1/2}\{z_2 - z_1 - (1 - Fr')^{1/2}[\Phi(z_2) - \Phi(z_1)] - (Fr')^{1/2}[F_1(z_2) - F_1(z_1)]\}$$

where $\Phi(z) = \int dz/(1 - z^n) + C$, $F_1(z) = \int dz/(1 + z^{n/2}) + C$, $t = t_2 - t_1$. $(\alpha B/g\omega)^{1/2}$ and Fr' are constant mean values for a given interval.

For falling and rising waves, and for $i \gtreqless 0$ author obtains six analogous equations.

The exponent n may be chosen arbitrarily, and hence it is possible to use for calculation of the integrals for $\Phi(z)$ and $F_1(z)$ the tables by Bakhmetyev for the index $x(= n)$ of the cross section of the bed.

Jan Smetana, Czechoslovakia

1625. Folsom, R. G., Determination of ASME nozzle coefficients for variable nozzle external dimensions, *Trans. Amer. Soc. mech. Engrs.* **72,** 5, 651–654, July 1950.

Report of experimental investigations on influence of the external dimensions (not included in specifications of the ASME Special Research Committee on Fluid Meters) of ASME long-radius nozzles on pressures measured at several points downstream of the nozzle flange. Discharge coefficients are almost

independent (less than $\pm 0.25\%$) of the clearance in annulus between the pipe wall and nozzle external diameter at all pressure-tap locations between holding flange and end of nozzle. At pressure-tap points downstream from the end, appreciable changes in the coefficient with the clearance exist, with larger differences corresponding to smaller diameter ratios.

Alb. Schlag, Belgium

1626. **Yao, T. P., and Kondic, V., Viscosity of metallic liquids,** *Nature* **166,** 4220, p. 483, Sept. 1950.

Little is known about the temperature-viscosity relation of liquid metals and alloys. Such information is of interest in theory of the liquid state, also in foundry practice. Experiments were made on lead, tin, and zinc-tin alloys. Diagram for pure tin and zinc-tin alloys given. Correlation between viscosity-temperature curve and composition is found. Results do not agree with some beliefs held previously. Experimental method is not described.

Peter Kyropoulos, USA

1627. **Veronese, Alessandro, Experiments with a hinged-leaf gate** (in Italian), *Energia elett.* **27,** 5, 270–275, May 1950.

Laboratory experiments were made on a hinged-leaf gate 40 cm wide and 50 cm high for the purpose of determining the forces and moments on the gate. These were measured directly with a dynamometer and by means of pressures along the gate. Results of these two kinds of measurements agree very well. The data are presented in graphs and tables.

V. A. Vanoni, USA

1628. **Pigott, R. J. S., Pressure losses in tubing, pipe, and fittings,** *Trans. Amer. Soc. mech. Engrs.* **72,** 5, 679–688, July 1950.

A novel method of analyzing elbow and bend losses is presented that correlates test data and yields practical formulas. Author considers bend or elbow loss made up of three elements: (*a*) Ordinary pipe friction; (*b*) minimum bend loss for given ratio radius of bend to diameter of pipe (for dead smooth conduit); (*c*) turbulence loss as function of relative roughness and Reynolds number. Loss *a* is computed with the pipe. By correlating loss data, author obtains the equation $\rho_1 = 0.106\,(r/d)^{-2.5}$ for loss *b*. Loss *c* is correlated to the Darcy-Weisbach friction factor for pipe *f* yielding $\rho_2 = 2000\,f^{2.5}$. The final formula for extra loss owing to the bend or elbow is $\rho = \rho_1 + \rho_2 = 0.106\,(r/d)^{-2.5} + 2000\,f^{2.5}$, where loss in feet of fluid flowing is the product of ρ and velocity head in pipe.

As equivalent length is given by $\rho d/f$, and the second term in the loss is a substantial part of the total loss, bend and elbow losses are conveniently tabulated under three parameters: type of pipe; diameter of pipe; and ratio of radius of bend to diameter of pipe.

Reviewer believes this paper is a substantial contribution to the difficult problem of minor losses.

V. L. Streeter, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 1688, 1734, 1828, 1836)

1629. **Scholz, N., On the calculation of the potential flow around airfoils in cascade,** *J. aero. Sci.* **18,** 1, 68–69, Jan. 1951.

Cascade of little-cambered airfoils are dealt with by the singularity method. This means that each airfoil of the cascade is replaced by a distribution of vortexes and sources situated on the median line of each profile. The field of induced velocity is to be calculated. To avoid the singularity of the integrand, it is convenient to evaluate the formula for induced velocity by subtracting that of an isolated airfoil:

$$w_{\text{casc}} - w_{\text{isol}} = (1/2t) \int_{-c/2}^{+c/2} g(x')[e^{i\lambda} \coth\,(\pi e^{i\lambda}(x - x')/t) - t/\pi(x - x')]dx'$$

Here λ is angle between cascade axis and y-axis, x direction of airfoil chord c, t distance of airfoils measured in cascade axis direction, and $g = q + i\gamma$ the distribution of sources and vortexes. The function in brackets is regular in the whole range of integration and is a universal function of $(x - x')/t$. It has been tabulated and shown in diagrams in *Z. angew. Math. Mech.*, enabling treatment of many examples of cascades, including calculated pressure distributions.

F. Riegels, Germany

1630. **Souriau, Jean-Marie, and Chastenet de Géry, Jérôme, Extension of the Küssner method to thick profiles** (in French), *Rech. aéro.*, no. 17, 9–15, Sept.-Oct. 1950.

Paper treats harmonic motion of two-dimensional airfoil of finite thickness in a perfect, incompressible fluid. Potential component ϕ_1 due to unsteady flow is treated as a small perturbation on steady-flow solution. Latter is represented by complex potential $\phi_0 + i\psi_0$. ϕ_0 is then posed as function of independent variables (ϕ_0, ψ_0) and satisfies Laplace's equation. Solution for ϕ_1 is via an early method of H. G. Küssner [*Luftfahrtforschung*, 1929]. Joukowsky airfoil with aileron is treated as an example. See also AMR **4,** Rev. 756.

John W. Miles, USA

1631. **McMillen, J. H., Kramer, R. L., and Allmand, D. E., Shadowgrams of spherical missiles entering water at supersonic speeds,** *J. appl. Phys.* **21,** 12, 1341–1342, Dec. 1950.

1632. **Van Deemter, J. J., Results of mathematical approach to some flow problems connected with drainage and irrigation,** *Appl. sci. Res. Sec. A*, **A2,** 1, 33–53, 1949.

Simple formulas are given for height of the water table in homogeneously, isotropically permeable land drained or irrigated by pipe drains, under different circumstances of rainfall or evaporation, emanation of artesian water, or seepage to great depth. In case of drainage by ditches, the formulas are complicated. Some numerical values are given. It is shown that Southwell's relaxation method can be applied to more general flow problems, particularly where the permeability is not homogeneous.

From author's summary

1633. **Polubarinova-Kochina, P. Ya., On sources and sinks on a surface** (in Russian), *Prikl. Mat. Mekh.* **14,** 1, 57–64, Jan.-Feb. 1950.

Paper deals with determination of the velocity potential and stream function for steady, incompressible, irrotational flow of a fluid on a surface due to a source (or sink) on the surface. The surface is supposed given in parametric form: $x = f_1(\alpha, \beta)$, $y = f_2(\alpha, \beta)$, $z = f_3(\alpha, \beta)$. If the parameters α and β are isothermal, that is

$$ds^2 = \lambda(\alpha, \beta)[d\alpha^2 + d\beta^2],$$

and if further $\lambda(\alpha, \beta) \equiv 1$, then the velocity potential φ and the stream function ψ of a flow on the surface due to a source at the point (α_1, β_2) both satisfy Laplace's equation $(\partial^2 u/\partial\alpha^2) + (\partial^2 u/\partial\beta^2) = 0$, and hence

$$\varphi(\alpha, \beta) = \tfrac{1}{2} \log\,[(\alpha - \alpha_1)^2 + (\beta - \beta_1)^2];$$
$$\psi = \text{arc tan}\,[(\beta - \beta_1)/(\alpha - \alpha_1)].$$

This remark is employed to determine φ and ψ for certain cylindrical surfaces and surfaces of revolution, and for a general ellipsoid.

Courtesy of Mathematical Reviews J. B. Diaz, USA

1634. Pai, S. I., On the stability for a certain degenerate type of disturbance of viscous fluid flow between parallel walls, *J. aero. Sci.* **17,** 8, p. 525, Aug. 1950.

Squire [*Proc. roy. Soc. Lond. Ser. A*, **142,** 621–8, 1933] showed that for parallel flow between walls there were no neutral, periodic, two-dimensional disturbances parallel to the walls (only finite R/λ seems to have been considered; here R is Reynolds number and λ wave length in the flow direction). Since the flow is stable for $R = 0$, it is stable for all finite R. This note proves stability directly (assuming finite R/λ implicitly) by a slight modification of Squire's method.
A. F. Pillow, Australia

1635. Foote, J. R., and Lin, C. C., Some recent investigations in the theory of hydrodynamic stability, *Quart. appl. Math.* **8,** 3, 265–280, Oct. 1950.

Paper is based on an M.I.T. thesis of Foote's and deals with problems of free boundary layers in parallel flows such as jets and wakes, where no solid boundary is present. In contrast to other stability problems it is shown that, by extension of conclusions reached in the infinite jet, effect of viscosity enters the stability problem for the jet profile only through higher approximations to the inviscid solutions of the stability equation. Symmetric and unsymmetric jet profile are treated separately; in the second case there are two critical points. Integration of the inviscid equation (essentially the vorticity equation for a perfect fluid) yields Reynolds shear stress and its rate of change along the real axis; for neutral disturbances there is a discontinuity at the critical points. A necessary condition for existence of self-excited disturbances is given. In an additional section Lin applies these results to the motion of a thin layer of fluid over a spinning body of revolution (instability of zonal winds on the earth).
H. Bilharz, Germany

1636. Goland, Leonard, A theoretical investigation of heat transfer in the laminar flow regions of airfoils, *J. aero. Sci.* **17,** 7, 436–440, July 1950.

Author presents a mathematical analysis of heat transfer in the thermal boundary layer about a certain infinite cylinder having a chordwise pressure gradient. Fluid is assumed incompressible, having constant properties, and negligible dissipation. Under usual boundary-layer order-of-magnitude argument, the energy equation for the temperature, with unit Prandtl number, is shown to be identical to the momentum equation for the spanwise velocity distribution in the laminar boundary layer of a certain yawed infinite cylinder in steady flow. Author uses the velocity solution of Sears [AMR **1,** Rev. 483] for this case, to solve for the temperature distribution in the boundary layer and heat loss from the isothermal surface. Results are compared with several available approximate methods for calculating heat loss from airfoils, and best agreement in this particular case is found for the method of Squire [*Report No. Aero. 1783, Roy. Aircr. Establishment*, 1942]. General conclusions regarding the relative accuracy of these methods for arbitrary airfoils is not, and should not be, drawn from the results of comparison in present case.
Lawrence M. Grossman, USA

1637. Coombs, A., The translation of two bodies under the free surface of a heavy fluid, *Proc. Camb. phil. Soc.* **46,** part 3, 453–468, July 1950.

Author solves the two-dimensional problem of lift and drag when two cylinders are submerged at a finite depth below the free surface of an otherwise undisturbed steady stream of incompressible inviscid fluid. The solution proceeds by mapping on two complex planes and use of the theorem of Blasius. Some tabulated numerical results are given for the case of two equal circular cylinders without circulation when (1) the centers are at depths $4l$ and $8l$ in the same vertical, and (2) the centers are each at depth $4l$ and distant $4l$ apart. L. M. Milne-Thomson, England

1638. Kerr, S. Logan, Surge problems in pipe line—soil and water, *Trans. Amer. Soc. mech. Engrs.* **72,** 5, 667–668, July 1950.

Paper describes the work done by the Water Hammer Committee of the ASME, outlining the publications involved. A general review and summary are given of water-hammer studies as they can be applied to the petroleum pipe-line surge problem.
From author's summary

1639. Moody, Lewis F., Some pipe flow characteristics of engineering interest and an approximate method of discharge measurement (in French), *Houille blanche* **5,** 3, 313–325, May-June 1950.

1640. Nikuradse, J., Laws of flow in rough pipes, *Nat. adv. Comm. Aero. tech. Memo.* 1292, 62 pp., Nov. 1950.

Translation from *VDI-Forschungscheft*, 361 (suppl. to *Forschung. Geb. Ing.-Wes.* Ausg. B, Bd. 4, July-Aug. 1933).

1641. Miyata, Y., On the deformation of colliding water drops (in Japanese), *J. meteor. Soc. Japan* **28,** 2, 71–75, Feb. 1950.

Under the assumption that colliding liquid drops moving relative to each other in air do not coalesce and that their deformation is small, the shape of the drop after collision with another drop of different radius is determined theoretically as a function of their radii, relative velocity, surface tension of the liquid, density of the air. The velocity potential is written for two spherical drops moving at velocities U and U' along the x axis and the differential equation governing the deformation is derived. A solution of the linearized equation is obtained. A. M. Kuethe, USA

1642. Brun, Edmond, and Vasseur, Marcel, Mechanics of suspensions in the case of a cylinder (in French), *Proc. seventh int. Congr. appl. Mech.* **2,** part I, 264–279, 1948.

Authors study the motion of small particles in a two-dimensional vortex flow around a sink. This flow pattern is characterized by a constant angle between the local velocity and the local radius vector from the sink as center. Flow lines are logarithmic spirals. Particles carried by this flow are considered to be small if local flow pattern around them is always laminar. Outside forces on both fluid and particles are neglected. The basic differential equation leads to a particular solution with concentric circular paths which is stable, and real for particle densities above a limiting value, which depends on the flow condition. All other trajectories run tangentially into this particular path in such a way that all particles have the tendency to accumulate at their particular distance from the center. Application of this peculiar flow pattern to sediment separators is suggested.
H. A. Einstein, USA

1643. Strang, J. A., Superposable fluid motions, *Comm. Fac. Sci. Univ. Ankara* **1,** 1–32, 1948.

Author calls two solenoidal velocity fields v_1 and v_2 satisfying Navier's equation for the motion of a viscous incompressible fluid "superposable" if their vector sum $v_1 + v_2$ is also a solution subject to suitable pressure and force fields. He calls a field v "self-superposable" if $2v$ is similarly a solution. He derives the condition curl $(v_1 \times w_2 + v_2 \times w_1) = 0$, where w_1 and w_2 are the respective vorticities, as necessary and sufficient for superposability, and the condition curl $(w \times v) = 0$ for self-superposability. In particular, he notes that irrotational and Beltrami motions $(w \times v = 0)$ are self-superposable, and that if two motions are such that the streamlines of either coincide with the vortex lines

of the other, they are superposable. [Reviewer's note: the author's analysis of self-superposable motions could have been simplified by using the fact that any circulation-preserving motion with steady vorticity is self-superposable, and that for this class of motions of viscous fluids the main theorems of the theory of inviscid fluids remain valid.] The author notes that the classical steady motion theorems of d'Alembert (w = constant on each streamline in plane motion) and Lamb (existence of Bernouillian surfaces) thus appear as statements that steady motions of inviscid incompressible fluids subject to conservative forces are always self-superposable. If v_1 is irrotational, a class of rotational flows superposable upon it can always be found; if curl curl $v_1 = 0$, a class of superposable irrotational flows can be found. The author gives special attention to plane motions, and shows that Rankine's construction of streamlines by superposition is valid only for irrotational flows or for flows in which $w = k\psi$, where ψ is the stream function and k has the same value for each. In a self-superposable flow the vorticity is propagated in accordance with the ordinary diffusion equation $\partial w/\partial t - \nu\nabla^2 w = 0$. In the remainder of the paper the author discusses special cases. He finds some new exact solutions, and states that most of those already known are self-superposable. C. Truesdell, USA

1644. **Davies, C. N., Viscous flow transverse to a circular cylinder,** *Proc. phys. Soc. Lond.* **63,** part 4, 364 B, 288–296, Apr. 1950.

Paper revises Lamb's solution for viscous flow past a circular cylinder using Oseen's method of linearization of the inertia terms in the Navier-Stokes equations. More rigorous results are obtained for the velocity field and resistance. Results are applicable at very small values of the Reynolds number.

M. J. Thompson, USA

1645. **Ruden, P., Two-dimensional symmetrical inlets with external compression,** *Nat. adv. Comm. Aero. tech. Memo.* 1279, 48 pp., Mar. 1950 (transl. *ZWB Forschungsber.* no. 1209, Apr. 15, 1940).

Diffusers with pressure conversion wholly within the duct are generally inefficient because of internal boundary-layer separation. On the other hand, diffusers with the compression taking place entirely in the external flow are free of this difficulty, but they require rather thick walls. In order to investigate the magnitude of wall thickness required for the latter case, author investigates simple inlets with constant velocity within the duct. Hodograph method is used with the assumption that fluid is incompressible and that inlets are semi-infinite in extent. Examples are computed showing detailed properties of the resulting inlets in the physical plane. H. Yoshihara, USA

Compressible Flow, Gas Dynamics

(See also Revs. 1629, 1701, 1703, 1704, 1716, 1722, 1727, 1729, 1730, 1731, 1737, 1748, 1749, 1756, 1760, 1771, 1772, 1784, 1785, 1803)

1646. **Wang, Chi-Teh, and Chou, Pei-Chi, Application of Biezeno-Koch method to compressible fluid flow problems,** *J. aero. Sci.* 17, 9, 599–600, Sept. 1950 (Readers' forum).

1647. **Cole, Boris N., Scale analysis of compressible-gas flow,** *Engineering* **170,** 4410,113–114, Aug. 1950.

Author claims that for compressible flows the conditions for dynamic similarity between geometrically similar flow models are equal Mach numbers and equal Reynolds numbers. The effect of variation of viscosity of the fluid with temperature, according to the Sutherland equation, is investigated on the basis of a scale analysis. A discussion of the use of this analysis for study of model and prototype gases, using helium and air, respectively, is given in terms of errors in pressure and temperature measurements. Joseph Kaye, USA

1648. **Siegel, Keeve M., Boundaries of fluid mechanics,** *J. aero. Sci.* **17,** 3, 191–192, Mar. 1950.

Comparison is made of various criteria for determining the regions of gas dynamics, slip flow, and free molecule flow.

H. Yoshihara, USA

1649. **Martin, John C., The calculation of downwash behind wings of arbitrary plan form at supersonic speeds,** *Nat. adv. Comm. Aero. tech. Note* 2135, 47 pp., July 1950.

Integrals for downwash in linearized supersonic flow have been presented in various forms in previous papers by Lagerstrom, Heaslet and Lomax, and others; use of vortexes for calculating downwash has been suggested by Robinson [AMR **2,** Revs. 1418, 1419] and exploited by Mirels and Haefeli [AMR **3,** Rev. 2404].

Author rederives various results in more unified form, discusses singularities of downwash integral and their treatment by Hadamard's finite-part concept, and presents some results in more convenient form for computations. Paper gives potential of lifting lines (thus facilitating use of vortexes in cases with spanwise discontinuity in lift and for sidewash calculations). Exact (integration) and approximate (summation) methods are compared. Paper confirms lifting-horseshoe-vortex method to be convenient, accurate means for computing downwash in many cases, if limitations of linearized theory—which neglects rolling up of vortex sheet—are acceptable. F. W. Diederich, USA

1650. **Ferri, Antonio, Supersonic flow around circular cones at angles of attack,** *Nat. adv. Comm. Aero tech. Note* 2236, 30 pp., Nov. 1950.

The known solution of Stone [AMR **1,** Rev. 993] and Kopal [*M.I.T. tech. Rep.* no. 3] assumes a cosine variation of entropy around a slightly inclined cone. Author shows entropy is actually constant over surface, and is multivalued at top of cone where streamlines converge, but changes rapidly to the Stone-Kopal distribution outside a thin "vortical layer" surrounding the cone. In first-order theory, which neglects squares of α (angle of attack), this layer has infinitesimal (order α^2) thickness, but across it the velocity, density, and entropy show first-order discontinuities. These are calculated, and hence the surface pressure.

Reviewer notes correct surface pressure is more easily calculated by ignoring effects of vortical layer. First-order Stone-Kopal results are unaffected because pressure is constant through vortical layer. It would be of interest to know whether second-order Stone-Kopal values of pressure [AMR **3,** Rev. 935] must be modified.

Author achieves good agreement between experiment and first-order theory up to $\alpha = 12°$ by retaining certain second-order (α^2) terms in calculating pressure. Reviewer notes this can equally well be done (with identical results) using first-order Stone-Kopal theory, and he believes second-order Stone-Kopal theory has been misinterpreted by author. Correct values agree with author's experiments even more closely than do his theoretical data. M. D. Van Dyke, USA

1651. **Thiruvenkatachar, V. R., Compressible shear flow. I. Thin airfoil in steady supersonic flow. II. Nonsteady motion of thin airfoil in supersonic flow,** *Proc. nat. Inst. Sci. India* **15,** 219–231, 233–240, 1949.

Problem is that of steady flow over a thin airfoil near the X-axis when the undisturbed velocity is in the X-direction and is a linear

function of y, the shear flow. The velocity of sound in the undisturbed flow is taken to be a constant. Elementary solutions of the linearized equations of motion near the X-axis are first obtained when the Mach number near the X-axis is greater than unity. These solutions are then superimposed to obtain the solution for the airfoil. From these, the lift and drag of an airfoil are calculated in terms of the airfoil profile. In the opinion of the reviewer, the author's choice of elementary solutions for the airfoil problem is arbitrary as he has not considered the behavior of the solution far away from the airfoil. Until this is checked, the results cannot be accepted.

Author gives an integral formula for the linearized velocity potential of two-dimensional unsteady shear flow over a thin airfoil by a technique developed by Taussky ["Courant Anniversary Volume," New York; Interscience Publishers, 1948, pp. 421–435; AMR **3**, Rev. 936] for case of uniform undisturbed flow, i.e., by using the elementary solution. However, as in author's previous paper, result is doubtful since he has not considered the conditions at infinity. H. S. Tsien, USA

1652. Harmon, Sidney M., and Jeffreys, Isabella, Theoretical lift and damping in roll of thin wings with arbitrary sweep and taper at supersonic speeds. Supersonic leading and trailing edges, *Nat. adv. Comm. Aero. tech. Note* 2114, 49 pp., May 1950.

Analysis based on linearized theory. Paper represents an extension of a previous analysis limited to plan forms having subsonic leading edges and supersonic trailing edges. Results presented in form of generalized design curves. Arthur L. Jones, USA

1653. Van Dyke, Milton D., A study of second-order supersonic-flow theory, *Nat. adv. Comm. Aero. tech. Note* 2200, 73 pp., Jan. 1951.

Substitution of the solution of the linearized equation of supersonic flow into the nonlinear part of the exact equation leads to the nonhomogeneous wave equation. This equation is solved completely for plane flows and flows over simple bodies of revolution, leading to results in excellent agreement with known exact solutions. For nonsymmetric three-dimensional flows a complete solution is found only for slightly yawed cones. Comparison with usual series expansion in powers of thickness parameter indicates in some cases a definite superiority of the method. R. E. Street, USA

1654. Meyerhoff, Leonard, An extension of the theory of the one-dimensional shock-wave structure, *J. aero. Sci.* **17**, 12, 775–786, Dec. 1950.

Author presents results of investigation of the one-dimensional steady shock-wave flow with two variations: constant, and variable gas properties. As the problem is interesting from an engineering standpoint, the numerical methods used (M.I.T. differential analyzer) represent only some extensions of previous known methods. Where gas properties are constant it is shown that there is an infinite number of shock-wave type solutions and only one of them terminates in a region of uniform flow with the values of Rankine-Hugoniot. Author points out an extreme sensitivity of the method used to numerical calculations and that even a highly accurate numerical solution may originate wrong conclusions.

The solution of the problem with temperature-dependent gas properties for the case of uniform exit flow is obtained by a graphical iteration procedure. To this end the limit values of Rankine-Hugoniot for variable gas properties are calculated. Results of a numerical example permit author to conclude that the shock wave is approximately two and one-half times greater than the mean free molecular path and that Navier-Stokes relations are valid for viscous shock-wave problem within certain limits.

Without any attempt to decrease the value of the results of such elaborate and lengthy calculations, one may make a few remarks. The fact that infinitely many solutions appear means, among other things, that assumed boundary conditions do not define the problem in a unique way. Hence, one questions whether the proposed methods (previous and the discussed one) utilizing the inflection point of the velocity are the "natural" approach. Certain other considerations seem to indicate that perhaps the inflection point of the pressure should be assumed as the center of the shock wave. Author does not discuss the pressure and density pattern in the shock. From the mathematical standpoint nothing is said about the nature of the graphical iteration process (convergence, asymptotic behavior). Reviewer believes that this point should be attacked by the strongest possible mathematical tool since all those methods may be highly sensitive to numerical calculations and may lead very easily to wrong conclusions. Author uses sonic speed and Mach number based on isentropic relation, although in viscous flow the velocity of sound is expressible by a more complicated relation. This point is not explained. Summarizing, author presents an interesting generalized theory of the shock phenomenon but does not present a deeper mathematical discussion of it, satisfying himself with approximate methods of solution. M. Z. Krzywoblocki, USA

1655. Thompson, M. J., A note on the calculation of oblique shock-wave characteristics, *J. aero. Sci.* **17**, 11, p. 744, Nov. 1950.

In this short note author outlines a method of calculating the characteristics of a plane oblique shock wave attached to a wedge in two-dimensional supersonic flow without elaborate charts and cross plotting. The wave angle θ_w is found by solving by numerical methods a cubic equation in $\sin^2 \theta_w$ from which θ_w is obtained in terms of wedge angle, Mach number, and specific heat ratio directly. Walker Bleakney, USA

1656. Cowan, George R., and Hornig, Donald F., The experimental determination of the thickness of a shock front in a gas, *J. chem. Phys.* **18**, 8, 1008–1018, Aug. 1950.

Equations are developed by which the density profile of a shock front may be calculated from a knowledge of its reflectivity as a function of wave length and angle of incidence. The reflectivities of shock front have been measured in nitrogen at various pressures, and from their change with wave length the thickness of the fronts was calculated with an accuracy of 25%.

From authors' summary by A. Petroff

1657. Cabannes, Henri, Calculation of the curvature at the apex of the attached shock wave in axially symmetric flow (in French), *C. R. Acad. Sci. Paris* **231**, 5, 325–326, July 1950.

Results obtained for conical flow are extended by means of the method of characteristics to include case of the shock wave formed in axially symmetric flow by a body whose radius of curvature at the apex is finite and not zero. Flow on both sides of the attached shock is assumed to be supersonic. Method is briefly outlined and pertinent equations are given but not derived. Ione Faro, USA

1658. Cabannes, Henri, On the attached shock wave when the velocity down-stream of the peak of the obstacle is subsonic (in French), *C. R. Acad. Sci. Paris* **230**, 21, 1830–1832, May 1950.

In plane supersonic flow past a sharp-nosed airfoil, at angles of attack such that the shock wave is attached but the velocity

directly behind it on the upper surface at the leading edge is subsonic, the flow conditions near the leading edge are considered. It is shown that the pressure and streamline deflection depart from their values at this point to a first approximation by quantities associated, according to the Prandtl-Glauert laws, with the velocities in the incompressible potential flow in a corner.

M. J. Lighthill, England

1659. **Oswatitsch, Klaus, Shock waves in stationary flow around flat profiles** (in German), *Z. angew. Math. Mech.* **29**, 129–141, 1949.

Paper begins with a conventional treatment of weak shock waves in stationary flow, including the calculation of bow- and stern-wave shapes for finite wedges and slender convex contours. It is shown how the drag can be evaluated by computing the entropy rise across the shock waves thus approximated. Identification of this drag with the usual pressure drag is made. The velocity and pressure distributions in the wake can be computed from the entropy distribution. Turning now to slightly supersonic flow, author assumes irrotational-isentropic flow and obtains approximate formulas for the characteristics and weak shock waves in this region. A rough procedure is suggested for estimating the shock drag of an airfoil with a local supersonic region, in a subsonic stream. This does not involve any considerations of boundary-layer separation; nevertheless, author claims that it explains the sharp drag-rise with increasing speed in the transonic region.

W. R. Sears, USA

1660. **Wang, Chi-Teh, and Rao, G. V. R., A study of the nonlinear characteristics of compressible flow equations by means of variational methods,** *J. aero. Sci.* **17**, 6, 343–348, June 1950.

Steady two-dimensional potential flows of compressible fluid about several bodies are solved by variational methods. If powers of the undetermined parameters higher than the second are neglected in the variational integral, a linearized solution is obtained which is shown to differ only slightly from the nonlinear solution. Saving of labor entailed is tremendous. For subsonic flow around a circular cylinder, the nonlinear solution with two parameters yields a unique symmetrical flow for all $M_0 < 0.537$, two symmetrical flows for $0.537 < M_0 < 0.615$, and no symmetrical flow possible for $M_0 > 0.615$. An unsymmetrical flow is possible for $M_0 > 0.51$. This behavior is taken to be a qualitative indication that there must be a breakdown of potential flow with the possible occurrence of shock waves.

Stephen H. Crandall, USA

1661. **Thomas, T. Y., The determination of pressure on curved bodies behind shocks,** *Comm. pure appl. Math.* **3**, 2, 103–132, June 1950.

Author assumes a steady, plane, supersonic, isentropic flow, potential in front of the shock and rotational behind it. Inclination of boundary of obstacle is a monotonically decreasing or increasing function of the arc length. To determine the pressure along boundary of the body behind the attached shock (so attractive a problem from the engineering standpoint), author represents pressure near the boundary by a function of the inclination of the streamlines ω and entropy S. That function is expanded into a power series in S with coefficients depending on ω. When the interaction between shock line and boundary is expressed, the above series yields an ordinary differential equation of infinite order for the determination of pressure along the boundary. By terminating this series after the sum of its first n terms, author obtains a differential equation of finite order for the nth approximation of pressure on the boundary. Explicit equations for first, second, and third approximations are given. No comparison with tests are mentioned.

Reviewer believes that on the front part of the body, agreement with test data may be fair, but when the influence of the boundary layer becomes stronger further from the vertex, the theory may fail due to the neglect of viscosity effects (transition and separation).

M. Z. Krzywoblocki, USA

1662. **Robinson, A., Wave reflection near a wall,** *Coll. Aero. Cranfield Rep.* 37, 25 pp., May, 1950.

The field of flow due to a shock wave or expansion wave undergoes a considerable modification in the neighborhood of a solid wall, in addition to the actual reflection at the wall. In the model proposed by author, a solution of the potential equation in the form of a trigonometric series in the uniform supersonic stream is combined with a solution in the form of a power series in the flow close to the wall. To obtain a physically correct answer, author traces the gradual evolution of the disturbances on passing through various layers (super-, trans-, subsonic). Final equations of incoming and outgoing waves are given in form of Fourier integrals. In the calculation of numerical examples, far-reaching simplifications are introduced and the obtained reflected disturbance is propagated upstream in the boundary layer only for a few multiples of the boundary-layer thickness. Experimental evidence shows the resulting disturbances clearly distinguishable at points located fifty or sixty times the boundary-layer thickness upstream from incoming wave. In trying to account for the discrepancy, author notes that basic assumptions may be inadequate in three respects: they neglect viscosity, vorticity, and involve linearization. Reviewer believes, also, that perhaps much stronger mathematical methods should be used.

M. Z. Krzywoblocki, USA

1663. **Christianovich, S. A., and Yuriev, I. M., Subsonic gas flow past a wing profile,** *Nat. adv. Comm. Aero. tech. Memo.* 1250, 29 pp., July 1950.

Using the hodograph transformation of the equations of plane compressible irrotational flow, Chaplygin obtained the equations $\varphi_s = (K)^{1/2}\psi_\vartheta$ $\varphi_\vartheta = -(K)^{1/2}\psi_s$ for the relation between velocity potential φ, stream function ψ, and the variables s and ϑ, where s is a function of the velocity magnitude and ϑ denotes the velocity direction. These equations reduce to the ordinary Cauchy-Riemann equations if, instead of the variable K (depending on the velocity) a constant, i.e., its value for the undisturbed flow in infinity, is inserted. Then $F = \varphi + i\varphi = \varphi + iK_\infty{}^{1/2}\psi$ is a function of the complex variable $s - i\vartheta$, as was remarked by Tsien, who was able to solve the problem of subsonic compressible flow past an arbitrary profile in this approximation, using the theory of complex functions in the absence of circulation. Authors solve the problem with circulation by considering F as the complex potential of a flow round the unit circle in a ξ plane and comparing this flow with another potential flow about the same circle, which results from a transformation of an incompressible flow in a $z = \mu + i\nu$ plane, which has the velocity vector $e^{s-i\vartheta}$. By adjusting the circulation constants in both circular flows it is possible to construct a flow about a closed profile in the physical $x + iy$ plane, pertaining to a certain flow about a given profile in the $\mu + i\nu$ plane. The transformation from a regular contour in the $\mu + i\nu$ plane to the contour in the $x + iy$ plane introduces certain singularities into this contour, which can be eliminated by means of an auxiliary transformation. Process is applied first to a circular arc in the μ, ν plane and to an ellipse. The transformed contour in the physical plane appears to differ only very little from the original contour and even the singularities, introduced by omission of the auxiliary transformation, have no serious bearing on the results of the calculation. Finally, the validity of Joukowsky's theorem for subsonic compressible flow is shown and expres-

sions for the lift and moment are obtained. Paper has some resemblance with Lin's paper on this subject [*Quart. appl. Math.* **4**, 1946, p. 291] but the variable $s - i\vartheta$ is different from the complex variable used by Lin. R. Timman, Holland

1664. Guderley, G., and Yoshihara, H., An axial-symmetric transonic flow pattern, *Quart. appl. Math.* **8**, 4, 333–339, Jan. 1951.

Solution of compressible flow with free Mach number one, so significant from the engineering standpoint, is extremely difficult. One cannot expect to have all the phases of such a flow solved in form of simple expressions. Paper is a nice contribution to elucidation of peculiar mathematical characteristics of such a flow. Mathematically, incompressible flow around a body is considered to possess a doublet at infinity. Authors solve the following problem: What type of singularity (called basic singularity) at infinity must the flow in question possess in order to create a pattern around a body of revolution? Starting from a particular solution of the nonlinear equation of sonic flow (with the usual approximations based upon transonic similarity law), by means of interesting transformations and utilizing a group property, authors obtain a first-order equation with an unknown, for the time being, parameter n. A discussion of singularities of that equation leads to conclusion that singular points of interest are saddle or nodal points or combination of both types. Selection of proper singular points enables authors to find the (one) value of n. This is achieved by means of numerical computations (not outlined). Next, authors show the streamline pattern for flow represented by the basic singularity, pressure coefficient distribution, and the streamline deflection. An approximate construction of a body shape which produces a flow at infinity given by the (found) basic singularity has been carried out in another paper and only the resulting contour is shown in present note. Authors do not mention how theoretically, at least, other body shapes can be obtained. M. Z. Krzywoblocki, USA

1665. Spreiter, John R., Similarity laws for transonic flow about wings of finite span, *Nat. adv. Comm. Aero. tech. Note* 2273, 21 pp., Jan. 1951.

Functional relations are derived for lift, moment, and drag which enable the correlation and extrapolation of experimental data in the transonic régime. If similitude in pressure distribution is to exist, two parameters must be held constant: first, $(1 - M_0^2)^{1/2}[(\gamma + 1)\tau]^{1/3}$ as for two-dimensional flows; and second, $(1 - M_0^2)^{1/2}A$, where A is the aspect ratio and τ the ordinate amplitude parameter. For instance, for a flat plate τ is the angle of attack, and for a symmetrical airfoil τ is the thickness ratio.

Similarity laws are derived by considering the equations of transonic small-perturbation potential theory. Perturbation velocities are taken as the difference between local velocities and the critical velocity of sound. The potential equation and boundary conditions are transformed into a new system by applying stretching factors to the coordinate axes, perturbation potential, critical speed of sound, and the ratio of specific heats. Similarity laws are then developed by specifying that the new system satisfy the same perturbation potential and nondimensional boundary conditions as the old system.

Author shows that for wings of infinite aspect ratio the similarity laws reduce to the form given by von Kármán. Thus, at sonic speed the lift of thin, flat airfoils is proportional to the two-thirds power, and the drag to the five-thirds power of the angle of attack. For thin, flat wings of vanishing aspect ratio, however, the more familiar relationships of the lift being proportional to the first power, and the drag to the second power of the angle of attack are found.

Similarity laws for linearized subsonic and supersonic wing theory are rederived. It is well known that these laws contain an arbitrary parameter which permits the laws to be expressed in a wide variety of forms. It is shown that one of these forms coincides with the transonic similarity laws. This indicates that range of applicability of the laws is not restricted to the immediate vicinity of sonic speed. Vernon J. Rossow, USA

1666. Poggi, Lorenzo, The reflection of "rectilinear" waves in one-dimensional gas flows, *J. aero. Sci.* **17**, 12, 813–815, Dec. 1950.

Author gives two exact analytical solutions for one-dimensional, unsteady, adiabatic ideal-gas flow. First represents exactly a simple expansion wave moving into still gas; second has a disposable time-scale parameter and represents a good approximation to the reflection of first wave at a rigid wall. H. H. M. Pike, England

1667. Scherrer, Richard, and Gowen, Forrest E., Comparison of theoretical and experimental heat transfer on a cooled 20° cone with a laminar boundary layer at a Mach number of 2.02, *Nat. adv. Comm. Aero. tech. Note* 2087, 18 pp., May 1950.

Results of this investigation are important in problem of providing cooling for supersonic aircraft. The Reynolds number based on cone length was varied from 2.22×10^6 to 5.01×10^6 and surface temperature from 0.814 to 0.914 of the measured recovery temperature. Temperatures were measured in region lying between 16 and 82% of the cone length from tip; surface temperature was uniform only behind the 42% point. Heat-transfer rates were determined by measuring the temperature rise of the cooling medium, cold air, as it flowed along the inner side of the outer skin from front to rear of cone.

It was found that in the region between the 52 and 82% points the experimentally obtained values of $N_u/R_e^{1/2}$ agreed within the experimental accuracy with the value 0.49 calculated from the formula valid for a uniform surface temperature $N_u/R_e^{1/2} = (C_d R_e^{1/2}/2)P_r^{1/3}3^{1/2}$. The local Nusselt number N_u, and local Reynolds number R_e, were based on distance from the point under consideration to the cone tip and on fluid properties just outside the boundary layer. The Prandtl number P_r was assumed to have the value 0.72. For Mach number and surface temperatures of the tests, the factor $C_d R_e^{1/2}$ was chosen as 0.635. The presence of the factor $3^{1/2}$ makes the formula applicable to cones. Neal Tetervin, USA

1668. Chapman, Dean R., Airfoil profiles for minimum pressure drag at supersonic velocities—general analysis with application to linearized supersonic flow, *Nat. adv. Comm. Aero. tech. Note* 2264, 38 pp., Jan. 1951.

Main contribution of paper is to show that minimum drag two-dimensional airfoil (for given thickness or given structural criteria) in many cases may have a blunt trailing edge. Degree of bluntness depends upon base-pressure coefficient and, in general, thickness at trailing edge increases with airfoil thickness. All calculations are based on the linearized theory and results are plotted against base-pressure coefficient. Since little information is available on base-pressure coefficients, optimum airfoils are not completely determined by paper. General wordiness tends to obscure more important ideas. Keith C. Harder, USA

1669. Woolard, Henry W., A note on the subsonic compressible flow about airfoils in a cascade, *J. aero. Sci.* **17**, 6, 379–381, June 1950.

Recent trend toward use of the turbomachine in aircraft and other installations has stimulated interest in methods for predic-

ing the aerodynamic characteristics of airfoils in a cascade arrangement. Many papers dealing with incompressible flow about airfoil cascades have been published, and although most are rather laborious to apply, the problem can be considered as essentially solved. On the other hand, the effect of compressibility seems to have been neglected in the literature except for reference 2, which is applicable for only one blade angle ($\beta = 90°$). In this paper Mr. Woolard derives expressions which are useful for converting pressure or force coefficients based on mean-stream conditions to coefficients based on entrance- or exit-stream conditions with compressible flow over the cascades, thus helping to fill the last gap of required relations in this field.

E. A. Bonney, USA

1670. **Samoilovich, G. S., Calculations of hydrodynamical lattices** (in Russian), *Prikl. Mat. Mekh.* **14,** 2, 121–138, Mar.-Apr. 1950.

Author gives a general method of calculating the flow around a lattice of profiles when the flow around one profile is known. Consider a region G exterior to given lattice of unit circles with gap t in z-plane and region g exterior to given lattice of profiles L with gap t_1 in the ζ-plane. The gap t_1 and the form of the profile L are assumed to be arbitrary. The method consists in constructing a function which maps conformally the region G on the region g, assuming that the function which maps the region exterior to the unit circle about the origin into the region exterior to the single profile L is known. Let the infinities in both planes correspond. Then the mapping function can be written in the form $\zeta = az + P(z)$, where $P(z)$ has the following properties: (1) It is a periodic function of z with period it; (2) it remains bounded for $z \to \infty$; and (3) in any neighborhood of the origin, and, therefore, because of the periodicity of $P(z)$ in the neighborhood of any circle, the expansion $P(z) = \sum_{n=-\infty}^{\infty} a_n z^n$ is valid. Author obtains for ζ the expression

$$\zeta = az + (\pi/t)\sum_{n=0}^{\infty}((-1)^n/n!)a_{-(n+1)}\,(d^n/dz^n)\coth(\pi z/t) \quad [1]$$

where

$$a_{-n} = (1/2\pi i)^{-1}\,{}_c\!\int P(w)w^{n-1}dw$$

and where the integration is performed along a unit circle about the origin. Series [1] plays for calculation of hydrodynamical lattices same role as Laurent series in mapping region exterior to unit circle around origin on exterior of a single profile L.

For the complex potential $F(\zeta)$ of the lattice flow the expansion

$$F(\zeta) = (a\overline{W} - \lambda\Gamma/2\pi i)z(\zeta) + (\Gamma/2\pi i)\ln(1/\lambda)\sinh\lambda z(\zeta)$$
$$+ \lambda\sum_{n=0}^{\infty}((-1)^n/n!)a_{-(n+1)}\,(d^n/dz^n)\coth\lambda z(\zeta) \quad [2]$$

is obtained, where Γ is circulation around a single profile L. In limit case when $\lambda = (\pi/t) \to 0$, i.e., when $t \to \infty$, the mapping series [1] goes over into the corresponding Laurent series and [2] into the known expression for complex potential of flow past a single profile L.

A method of calculating the coefficients a and a_{-n} of the series [1] for given λ in terms of the coefficients c_1 and c_{-n} of corresponding Laurent series is then given.

As examples, calculation of potential flow through lattice of circles and through blades of a steam turbine is given.

E. Leimanis, Canada

1671. **Matthieu, P., The hydrodynamic significance of automorphic functions (plane flows around circular arc polygons)** (in German), *Comment math. Helv.*, **23,** 80–122, 1949.

Paper is concerned with the numerical computation of two-dimensional potential flows about profiles consisting of n circular arcs. While it has been known for a long time that the required conformal mappings are given by automorphic functions which can be represented in each case as the ratio of two independent solutions of a certain Fuchsian differential equation of second order, this has not been of much practical help since, for $n > 3$, the differential equation contains unknown parameters which depend on the geometry of the curvilinear polygon in an indirect way and are not known a priori. Author suggests overcoming this difficulty by integrating the differential equation with "well chosen" values of the unknown parameters and determining polygon so obtained. Repeating this process often enough with varied values of the parameters, one then tries to obtain a polygon close to the desired shape. Some very simple examples are treated numerically without, however, any attempt to estimate the effect of quality of convergence of the various infinite processes involved upon the reliability of final geometrical shape obtained.

Courtesy of Mathematical Reviews Z. Nehari, USA

1672. **Martin, M. H., Plane rotational Prandtl-Meyer flows,** *J. Math. Phys.* **29,** 2, 76–89, July 1950.

Deals chiefly with class of flows author calls "general Prandtl-Meyer" (GPM) flows defined by properties: (1) $\theta = \theta(q)$, where θ is flow direction, (2) isobars and isoclines do not coincide, (3) isobars and streamlines do not coincide, and (4) the speed q, density ρ, and pressure gradient dp/ds along a streamline remain finite without vanishing. The analysis is based on formulation using p and ψ (stream function) as independent variables. Some results are obtained for quite general classes of gases; however, most significant result is for polytropic gases:

The only isoenergetic, anisentropic GPM-flows of a polytropic gas have hodograph curves satisfying $\theta'' - a(q)\theta' - b(q)\theta'^3 = 0$, with

$$a \equiv 2\lambda(q - q^{-1})^{-1} - q^{-3}[{}^1\!/_2(1 + q^{-2}) - l]^{-1}$$
$$b \equiv [{}^1\!/_2(q - q^{-1}) - (\lambda + l)q][{}^1\!/_2(1 + q^{-2}) - l]^{-1}$$

where $l \equiv \gamma(\gamma - 1)^{-1}$ and λ is real parameter determining entropy distribution which must be of form: $S(\psi) = (C_p/\lambda)\log|\psi - \psi_0| + \text{const}, \lambda \neq 0$; or $S(\psi) = c(\psi - \psi_0) + \text{const}, \lambda = 0$. Here C_p is specific heat at constant pressure and ψ_0, $C \neq 0$ are any real constants.

Detailed study of specific cases is reserved for a future paper.

R. C. Prim, III, USA

1673. **Kaplan, Carl, On the particular integrals of the Prandtl-Busemann iteration equations for the flow of a compressible fluid,** *Nat. adv. Comm. Aero. tech. Note* 2159, 16 pp., Aug. 1950.

Particular integrals of the second- and third-order Prandtl-Busemann iteration equations for two-dimensional subsonic flow are obtained in terms of the first-order velocity potential and its derivatives. A systematic procedure for obtaining the particular integrals of all orders is described. These integrals may be adapted to supersonic flows by a change in sign of one of the parameters involved.

E. B. Klunker, USA

1674. **Pinkel, I. Irving, and Serafini, John S., Graphical method for obtaining flow field in two-dimensional supersonic stream to which heat is added,** *Nat. adv. Comm. Aero. tech. Note* 2206, 62 pp., Nov. 1950.

Method involves step-wise computation of the pressure and Mach number changes and of waves generated by a given distribution of heat sources. Auxiliary charts are included which reduce the labor of calculation. Evaluation of usefulness and accuracy of method could be made by comparison with analytically calculable diabatic flows and with numerical integration of the hydrodynamic equations. [See *Ball. Res. Labs. Rep.* 720, July 1950, and references cited therein.] Bruce L. Hicks, USA

1675. Eggers, A. J., Jr., One-dimensional flows of an imperfect diatomic gas, *Nat. adv. Comm. Aero. Rep.* 959, 11 pp., 1950.

See AMR **3**, Rev. 523.

1676. Chapman, Dean R., Note on base pressure measurements, *J. aero. Sci.* **17**, 12, 812–813, Dec. 1950.

Author discusses certain differences which have been reported in the literature on base pressure of bodies of revolution in supersonic flow. He points out that, as a result of experimental and theoretical studies, four characteristics of base pressure are now well established. These are the effects due to (a) type of boundary-layer flow and Reynolds number, (b) changes in surface temperature, (c) body shape, and (d) presence of tail fins. On basis of these characteristics it is stated that (1) discrepancy between German wind-tunnel and flight measurements can be attributed to Reynolds number effects and type of boundary-layer flow; (2) apparent agreement of base-pressure coefficient of a sphere and a missile with tail fins has no significance; (3) agreement between Gabaud's theory and experimental base-pressure measurements on missiles with tail fins is fortuitous; (4) the knee of the data on finned missiles and the parameters f and λ, later introduced by Hill as a modification of the known von Kármán-Moore dynamical theory, "appear to be more characteristic of the particular tail geometry used rather than of some intrinsic behavior of base pressure in the range of Mach numbers between 1.5 and 2.0." The first two statements are quite plausible, but with regard to the last two there is considerable doubt since for (3) the Gabaud theory is not available in this country, and for (4) the significance of the parameters f and λ has not been correctly presented. F. K. Hill, USA

1677. Chapman, Dean R., Laminar mixing of a compressible fluid, *Nat. adv. Comm. Aero. Rep.* 958, 7 pp., 1950.

See AMR **3**, Rev. 548.

1678. Sewell, Geoffrey L., Theory of an oscillating supersonic aerofoil, *Aero. Quart.* **2**, part I, 34–38, May 1950.

Under assumption of weak, moving, curved shock of small deviation from uniform stream Mach wave, author shows that solution corresponds to solution of Temple and Jahn for a weak, stationary, straight shock. John A. Lewis, USA

1679. Thwaites, B., Note on the circulatory flow about a circular cylinder through which the normal velocity is large, *Quart. J. Mech. appl. Math.* **3**, part 1, 74–79, Mar. 1950.

It has been established experimentally that, using suction velocity in an otherwise still fluid, a circulation can be set up about a porous circular cylinder fitted with a flap. General solutions of the circulatory viscous flow are obtained in the form of asymptotic series in inverse suction Reynolds number, both for steady and unsteady cases. From these solutions, velocity and pressure fields and torque on the cylinder can be calculated. It is found that, for values of suction section appropriate to this asymptotic theory, the circulation at infinity remains constant.

C. T. Wang, USA

1680. Synge, J. L., Note on the kinematics of plane viscous motion, *Quart. appl. Math.* **8**, 1, 107–108, Apr. 1950.

Note proves the following theorem: A compressible viscous fluid moves inside a fixed connected boundary B, on which the velocity vanishes. The necessary and sufficient condition for the consistency of the equations $u_x + v_y = \theta$; $v_x - u_y = 2\omega$; $(u)_B = 0$, and $(v)_B = 0$ is that $\int(2\omega U - \theta V)dx\,dy = 0$, where U is an arbitrary harmonic function, V the conjugate harmonic function, u and v are the velocity components. C. T. Wang, USA

1681. Lagerstrom, Paco A., Cole, Julian D., and Trilling, Leon, Problems in the theory of viscous compressible fluids, California Institute of Technology, Guggenheim Aeronautical Laboratory, iv + 232 pp., Oct. 1950.

Reprint of a former report [AMR **2**, Rev. 1415]. A detailed discussion of the incompressible and compressible flow over a semi-infinite and a finite flat plate and a comparison with the conventional boundary-layer theory have been added.

Gottfried Guderley, USA

1682. Smith, R. V., Determining friction factors for measuring productivity of gas wells, *J. Petrol. Technol.* **2**, 73–82, Mar. 1950.

This is a "hydraulic" discussion of the steady flow of gas through a pipe. Friction, work of gas expansion, and gravitational potential energy are included. Author illustrates two computational procedures for using the "hydraulic" energy equation to determine friction factors. The first is a graphical integration; second is an analytic integration made possible by using effective (constant) values for some variables along the pipe.

Stanley Corrsin, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 1635, 1677, 1802, 1849, 1864)

1683. Batchelor, G. K., Diffusion in a field of homogeneous turbulence, *Austral. J. sci. Res. Ser. A*, **2**, 4, 437–450, Dec. 1949.

Paper considers the mechanism of diffusion in the simple case of a homogeneous field of turbulence. It is shown to be useful to distinguish between diffusion from fixed and moving centers, and only the former is considered here. The diffusion of a cloud of marked fluid particles about average position of their center is known when the statistical behavior of a single fluid particle is known. Theory shows that dispersion of a fluid particle about its initial position increases first as the square of the time of flight t, then more slowly, and eventually increases linearly in t. Several different experiments have shown that the probability distribution of the displacement of a fluid particle is normal for all values of time of flight. As a consequence of these two facts, it is possible to represent the diffusion by a differential equation of the heat-conduction type, with a diffusion coefficient which initially increases with t and eventually becomes constant. Some consequences of the analysis are presented.

From author's summary

1684. Townsend, A. A., The fully developed turbulent wake of a circular cylinder, *Austral. J. sci. Res. Ser. A*, **2**, 4, 451–468, Dec. 1949.

Extending previous work on turbulent diffusion in the wake of a circular cylinder, a series of measurements have been made of turbulent transport of mean stream momentum, turbulent energy, and heat in the wake of a cylinder of 0.159-cm diam, placed in an airstream of velocity 1280 cm/sec. It has been possible to extend the measurements to 950 diam downstream from cylinder, and it is found that, at distances in excess of 500 diam the requirements of dynamical similarity are very nearly satisfied. To account for observed rates of transport of turbulent energy and heat, it is necessary that only part of this transport be due to bulk convection by the slow large-scale motion of the jets of turbulent fluid emitted by the central, fully turbulent core of wake, which had been supposed previously to perform most of transport. Remainder of transport is carried out by the small-scale diffusive motion of the turbulent eddies within the jets, and may be described by assigning diffusion coefficients to the turbulent fluid. It is found that diffusion coefficients for momentum

and heat are approximately equal, but that for turbulent energy is considerably smaller. On basis of these hypotheses, it is possible to calculate the form of the mean velocity distribution in good agreement with experiment, and to give a qualitative explanation of the apparently more rapid diffusion of heat.

From author's summary

1685. **Breslin, John P., and Macovsky, Morris S., Effects of turbulence stimulators on the boundary layer and resistance of a ship model as detected by hot wires,** *David W. Taylor Mod. Basin Rep.* 724, NS 715-086, 40 pp., Aug. 1950.

Ship-model experiments cannot be conducted under conditions dynamically similar to those for full-scale ships. Since Reynolds number is always too low, the boundary layer on the model remains laminar for a greater distance from leading edge than on full-size ship. This makes the predicted resistance too low. Turbulence stimulators, consisting of trip wire, roughness strip, and rod towed ahead were investigated for a tanker model of 1/24 linear scale. Resistance tests were made, and hot-wire probes were used to detect turbulent flow. Hot-wire techniques, already proved in air, were found to be applicable to water for distinguishing between laminar and turbulent flow. Adaptation of the hot wire and the demonstration of its worth in water constitute the more important contributions of the work. Turbulence stimulators were found to have an effect in desired direction, but no definite conclusions could be drawn as to best type of stimulator, nor as to optimum technique of artificial stimulation.

G. B. Schubauer, USA

1686. **Rotta, J., The spectrum of isotropic turbulence in statistic equilibrium** (in German), *Ing.-Arch.* **18**, 60-76, 1950.

Author considers the power laws of decay and the spectrum of turbulence following Heisenberg's theory. The one-dimensional spectrum thus calculated (with turbulence energy varying inversely as time) is compared with measurements of Simmons and Salter with satisfactory agreement [see AMR **4**, Rev. 347].

C. C. Lin, USA

1687. **Batchelor, G. K., and Stewart, R. W., Anisotropy of the spectrum of turbulence at small wave-numbers,** *Quart. Mech. appl. Math.* **3**, part 1, 1-8, Mar. 1950.

In a previous paper [AMR **3**, Rev. 332] it has been shown theoretically that the tendency to isotropic turbulence does not exist for large-scale components of homogeneous turbulence. This paper provides experimental evidence that large-scale components of turbulence behind a grid in a uniform stream are anisotropic, even though total turbulent energy may be distributed with approximate spherical symmetry. Evidence consists (a) of a comparison between two different velocity correlations for large spatial intervals, in initial period of decay, and (b) of measurements of directional distribution of turbulent energy in final period of decay when all except large-scale components of motion have been greatly reduced by viscous decay.

From authors' summary

1688. **Civrac, J., and Sirieix, M., Research on boundary layers. Comparison of theory and experiment on profile S. O. 6008 bis** (in French), *Rech. aéro.*, no. 18, 45-53, Nov.-Dec. 1950.

The turbulent boundary layer on the surface of a high-speed airfoil section (S.O. 6008 bis) has been measured at Reynolds numbers of 2.75×10^6 and 5.5×10^6 in the large wind tunnel at Chalais-Meudon. Measurements of normal pressure, momentum thickness, and profile drag are compared with theoretical predictions of a method of Germain and Civrac, based on work of Gruschwitz-Wahl for the turbulent layer and Falkner for the laminar layer. Predictions of normal pressure are accurate except close to the trailing edge, but there are substantial differences between observed and calculated values of momentum thicknesses and profile drag. These differences are substantially due to assumption that transition occurs at point of maximum speed (minimum pressure), and artificially induced transition at a definite point leads to good agreement. Near the trailing edge, errors are due to the theoretical null velocity at the dihedral, and a more plausible distribution is necessary.

A. A. Townsend, England

1689. **Battin, R. H., and Lin, C. C., On the stability of the boundary layer over a cone,** *J. aero. Sci.* **17**, 7, p. 453, July 1950.

1690. **Borg, S. F., A note on boundary-layer type solutions in applied mechanics,** *J. aero. Sci.* **17**, 4, 253-255, Apr. 1950.

This note lists a number of engineering problems which are stated to be of boundary-layer type because certain terms in their governing equations may be neglected except in the immediate vicinity of boundaries.

D. N. de G. Allen, England

1691. **Wuest, W., Contribution to non-stationary laminar boundary on plane walls** (in German), *Ing.-Arch.* **17**, 3, 193-198, 1949.

Author studies the unsteady motion over an infinite wall moving parallel to itself and near the boundary of two fluids sliding past each other. In either case, equations of the type of heat conduction are obtained, and well-known results can be used. For example, solution can be given in a closed form when plate motion may be described as a polynomial in time. Results of the two-fluid problem are applied to wind blowing over water surface. It is stated that in another paper, present results will be used as a first approximation in discussion of the usual stationary boundary-layer problem (see following review).

C. C. Lin, USA

1692. **Wuest, W., Formation of a laminar boundary layer beyond a suction spot** (in German), *Ing.-Arch.* **17**, 3, 199-206, 1949.

Author considers flow over two semi-infinite plates, parallel to each other, and with leading edges displaced relative to each other by a distance x_0 parallel to the plates and a distance y_0 perpendicular to them. Suction between the plates is just sufficient to remove lower part of the boundary layer over the first plate. Boundary layer over second plate is then investigated by using results of above paper after approximating the boundary-layer equation in the von Mises form by the equation of heat conduction.

C. C. Lin, USA

1693. **Whitehead, L. G., and Canetti, G. S., The laminar boundary layer on solids of revolution,** *Phil. Mag.* (7), **41**, 321, 988-1000, Oct. 1950.

Methods developed by Piercy, Whitehead, and Tyler [AMR **2**, Rev. 1170] for calculating plane boundary layers are extended to bodies of revolution. The problem for cones is reduced to that for wedges, which has been solved numerically by Hartree. An approximate solution is developed for general bodies. The velocity in the boundary layer is approximated by two terms of a series expansion valid near the nose plus a universal function of distance through the boundary layer chosen to give the exact (Blasius) solution for a cylindrical tube. This function is multiplied by a parameter varying streamwise, which is determined from the momentum integral. The approximation shows good agreement with the accurate solution for cones and with Tomo-

tika's solution for a sphere. In comparison with experiments on a 3:1 ellipsoid it fails only near the laminar separation point.
M. D. Van Dyke, USA

1694. Betz, Albert, Series representation of velocity distribution in laminar boundary layers (in German), *Arch. Math.* **2,** 220–222, 1950.

Author suggests use of the function

$$1 - \exp\{-(b_1\eta + b_2\eta^2 + b_3\eta^3 + \dots)\}$$

to represent the velocity distribution in a laminar boundary layer, where $\eta = yU/(2\nu\int_0^x U dx)^{1/2}$, x and y are the distances along and perpendicular to the solid boundary, U is free-stream velocity, and ν is the kinematic viscosity coefficient. The coefficients b_1, b_2, b_3, ... are to be determined by the conditions for $\partial^n u/\partial y^n$ at $y = 0$. By comparing with the Hartree profiles, author concludes that three or four terms of the series give desired accuracy.
C. C. Lin, USA

1695. Wieghardt, K., On a new energy theorem for the calculation of laminar boundary layers (in German), *Ing.-Arch.* **16,** 3–4, 231–242, 1948.

Author employs an integral equation for the energy flux in an incompressible boundary layer as well as the integral form of the momentum equation and defines an energy defect thickness of the boundary layer in analogy to the conventional displacement and momentum thicknesses. This additional equation allows use of a two-parameter family of boundary-layer profiles in approximate solutions instead of the conventional one-parameter family. Parameters chosen are $\lambda^* = (\delta_2^2/\nu)du/dx$, $\epsilon = (\delta_2/U)\partial u/\partial y|_0$, where δ_2 is momentum thickness of boundary layer, U free stream velocity, u boundary-layer velocity parallel to surface, ν kinematic viscosity, and x, y coordinates parallel and normal to the surface, respectively. First parameter is conventional while the second represents a dimensionless form of the slope of the boundary profile at the surface and is consequently related to the shear distribution. Author discusses a numerical method for computing the variation of these parameters in the direction of flow when an initial boundary-layer profile and free-stream velocity distribution are given, which amounts essentially to a stepwise integration of the simultaneous differential equations resulting from the energy and momentum integral expressions.

An approximation using a polynomial

$$u/U = 1 - (1 - y/\gamma)^8(1 + A_1 y/\gamma + A_2(y/\gamma)^2 + A_3(y/\gamma)^3$$

is considered [see Mangler, *Z. angew. Math. Mech.* **24,** 251–256, 1940]; and a two-parameter analysis is carried out for cases of a linearly decreasing free-stream velocity, a circular cylinder, and an elliptic cylinder. Results compare favorably with more nearly exact solutions and experimental results.
Courtesy of Mathematical Reviews F. E. Marble, USA

1696. Walz, A., Application of Wieghardt's energy theorem to one-parametric velocity profiles in laminar boundary layers (in German), *Ing.-Arch.* **16,** 3–4, 243–248, 1948.

The integral form of the energy equation for the laminar boundary layer used by Wieghardt (see preceding review) is employed in the calculation of a single parameter family of velocity profiles. The introduction of this energy relation in addition to the integral form of the momentum equation permits author to delete the condition which follows from the boundary-layer equation:

$$\left|\frac{dU}{ds}\frac{\delta_2^2}{\nu} = -\frac{\partial^2(u/U)}{\partial(y/\delta_2)^2}\right|_{y=0}$$

where U and u are free-stream and local velocity components parallel to surface, δ_2 momentum thickness of boundary layer, ν kinematic viscosity and s and y coordinates parallel and normal, respectively, to the surface. A set of two first-order differential equations results and a convenient arrangement of their numerical solution is devised. Author applies his method to the boundary layer in an adverse pressure gradient resulting from a linear decrease of free-stream velocity parallel to surface and to boundary layer of an elliptic cylinder. Results are in fair agreement with those of Howarth and of Wieghardt.
Courtesy of Mathematical Reviews F. E. Marble, USA

1697. Frenkiel, F. N., On turbulent diffusion, *Symposium on turbulence, July 1, 1949, Naval Ord. Lab., White Oak, Md., Rep.* NOLR-1136, pp. 67–86, 1950.

The three-dimensional turbulent diffusion of particles suspended in a flowing fluid is studied by assuming the distribution of these particles to follow the error law when they are emitted from a source in a stationary fluid, the variance of the error-law distribution varying with time. Experiments are described for comparison with the theory.
C. C. Lin, USA

1698. Frenkiel, François N., Statistical study of turbulence. Spectral functions and correlation coefficients (in French), *Off. Nat. Ètud. Rech. aéro. Rep.* 34, 103 pp., 1948 (publication delayed since 1942).

In the first chapter, author emphasizes the difference between time- and space-correlations and spectra of turbulence as well as between the Lagrangian and Eulerian viewpoint. He gives some new relations for isotropic and homogeneous turbulence and discusses the validity of the relation between Taylor's time spectrum and the longitudinal space correlation. After having initiated the use of one-dimensional space spectra, author gives the relations between the longitudinal and transverse spectra. The discussion of three-dimensional spectra (introduced by reviewer and other authors after completion of this paper) is not included. In the following chapters, author shows how experimental correlations and spectra can be represented by empirical curves. Numerous functions and the conditions of their use are discussed. Von Kármán's equation is used to find analytically the transverse correlation when the longitudinal correlation function is determined and, reciprocally, the inverse equation given by author is used to compute analytically the longitudinal correlation curves from empirical transverse correlation functions. The spectral functions are computed analytically for the various correlation functions and examples of spectral and correlation curves are given in numerous curves.
J. Kampé de Fériet, France

1699. Deissler, Robert G., Analytical and experimental investigation of adiabatic turbulent flow in smooth tubes, *Nat. adv. Comm. Aero. tech. Note* 2138, 41 pp., July 1950.

Theoretical analysis constitutes slight extension of Prandtl-von Kármán treatment. Near wall, shear stress is assumed constant and equal to viscous shear stress plus turbulent shear stress, with eddy viscosity proportional to product of density by local velocity by distance from wall. Solution of this first-order differential equation gives velocity distribution near wall. Von Kármán's formulas based on similarity law and (1) uniform shear stress and (2) shear-stress proportional to radius are rederived. All formulas are then generalized for compressible flow by introducing static temperature variation and consequent variation in density (inversely proportional to static temperature) and viscosity (assumed proportional to power of the temperature).

New measurements of velocity distribution and friction on smooth tube, diameter 0.87 in., length 87 in., with rounded and

right-angle-edge entrances, Reynolds numbers 10,000 to 200,000, Mach numbers to 0.5. Data are not given in tabular form, but agree closely with other investigators and analysis of author. Constants are $C = 3.8$ and $\kappa = 0.36$ compared with Nikuradse's 5.5 and 0.40. Effect of compressibility on velocity distribution is small.

Hugh L. Dryden, USA

1700. **Shirogane, Zensaku, The decay of turbulence** (in Japanese), *J. Soc. appl. Mech. Japan* **2**, 25–26, 1949.

Law of the decay of isotropic turbulence is discussed by assuming that Maxwell's stress for a plastic body is applicable to Reynolds' stress. The mean square of the length of vortex diffusion and diameter of the minimum vortex are also discussed.

Courtesy of Mathematical Reviews T. Okamoto, Japan

1701. **Wilson, Robert E., Turbulent boundary-layer characteristics at supersonic speeds. Theory and experiment,** *J. aero. Sci.* **17**, 9, 585–594, Sept. 1950.

Presented theory is based on the assumption that von Kármán's differential equation for the velocity distribution $(\tau/\rho)^{1/2} = -\kappa U_y^2/U_{yy}$ applies for compressible flow (U is velocity in direction along flat plate, ρ is density). The shear stress τ is assumed to be independent of the distance y normal to plate. The constant κ is assumed to be unaffected by compressibility and equals 0.4. Taking the case of a thermally insulated flat plate and neglecting radiation effects, the boundary-layer velocity distribution and skin-friction coefficients have been calculated. This calculation gives mean turbulent skin-friction coefficients, which are considerably higher than those given by von Kármán's result [*Proc. fifth Volta Congress*, Rome 1935].

Theory is compared with turbulent boundary-layer characteristics at supersonic speeds, which have been experimentally determined from Pitot pressure surveys made at several stations along a flat plate. Observed discrepancies between theoretical and experimental boundary-layer velocity distributions for compressible flow are of same order of magnitude as those for incompressible flow case. Agreement between theoretical and experimental mean turbulent skin-friction coefficients is within the scatter of the test data. Boundary-layer shape parameter H, defined as ratio of displacement thickness to momentum thickness, may be calculated from a velocity distribution of the form $U/U_1 = (y/\delta)^{1/n}$, where, for most purposes, n may be assumed to be independent of Reynolds number and Mach number and equal to 7, and U_1 is the free-stream velocity.

The check between theory and experiment is limited by the maximum test values of Mach number and Reynolds number, which are approximately 2.2 and 19×10^6, respectively.

Julius Rotta, Germany

1702. **Ludwieg, H., and Tillmann, W., Investigations of the wall-shearing stress in turbulent boundary layers,** *Nat. adv. Comm. Aero. tech. Memo.* 1285 (transl.), 25 pp., May 1950.

See AMR **4**, Rev. 822.

1703. **Stalder, Jackson R., Rubesin, Morris W., and Tendeland, Thorval, A determination of the laminar-, transitional-, and turbulent-boundary-layer temperature-recovery factors on a flat plate in supersonic flow,** *Nat. adv. Comm. Aero. tech. Note* 2077, 20 pp., June 1950.

The temperature-recovery factor r as defined by $r = [(T_{aw} - T_1)/T_1][2/(\gamma - 1)](1/M_1^2)$ (where T_{aw} is absolute temperature of the adiabatic wall, T_1 local free-stream temperature, γ ratio of specific heats, M_1 Mach number) has been measured very carefully at a Mach number of 2.4 and a Reynolds number range from 0.2×10^6 to 6.7×10^6. r is almost independent of the Mach and Reynolds number and depends solely upon the Prandtl number. In the laminar part $r = 0.881$, i.e., 1.5% larger than $Pr^{1/2}$ when the air properties are evaluated at the tunnel free-stream temperature. In the turbulent part of the boundary layer r varied from 0.897 to 0.884 in the range of Reynolds number from 2×10^6 to 6.7×10^6. These are just the values computed from Squire's equation (for incompressible flow $r = Pr^{(1+n)/(3+n)}$ for an $n = 1/_6$-power-law velocity distribution, using air properties at the surface and at the free-stream temperature, respectively.

Karl Wieghardt, Germany

Aerodynamics of Flight; Wind Forces

(See also Revs. 1437, 1441, 1448, 1551, 1552, 1649, 1652, 1741)

1704. **Frick, C. W., and Chubb, R. S., The longitudinal stability of elastic swept wings at supersonic speed,** *Nat. adv. Comm. Aero. Rep.* 965, 12 pp., 1950.

See AMR **4**, Rev. 1281.

1705. **Hassell, James L., and Bennett, Charles V., The dynamic lateral control characteristics of airplane models having unswept wings with round- and sharp-leading-edge sections,** *Nat. adv. Comm. Aero. tech. Note* 2219, 26 pp., Nov. 1950.

For flight conditions investigated, no apparent effect of airfoil section on flying characteristics of model having high inertia in yaw was noted, even for low values of directional stability. The normal-inertia model equipped with the NACA 0012 wing showed an adverse yawing motion in aileron rolls which increased with reduced values of directional stability; when this model was equipped with the 12%-thick biconvex wing, however, almost no yawing occurred, even at lowest values of directional stability. Difference in yawing characteristics of the models with the two wings was attributed to the effect of C_{n_p}, the rate of change of yawing-moment coefficient with rolling-angular-velocity factor, which was adverse for the NACA 0012 wing and favorable for the 12%-thick biconvex wing.

From authors' summary

1706. **Goland, Martin, The quasi-steady air forces for use in low-frequency stability calculations,** *J. aero. Sci.* **17**, 10, 601–608, 672, Oct. 1950.

It is shown that the quasisteady (Q.S.) approximation to the exact oscillatory air forces is not correct even within first order of the reduced frequency k. Correct expression would contain a.o. in its imaginary part a term of order $k \ln k$ [see also Miles, title source, **16**, p. 440, 1949]. Due to this term the stability derivative $c_{n_{D\delta}}$ (moment on whole aircraft due to elevator rotational velocity) has for low k-values the opposite sign compared with Q.S. approximation. This is a serious discrepancy for stability calculations, since Q.S. approximation leads to a static divergence which is contrary to experience and which disappears when giving $c_{n_{D\delta}}$ the other (true) sign. To avoid the logarithmic term leading to transcendental stability equations instead of polynomial equations, author suggests new approximations which for the real parts of the air forces are the Taylor series about $k = 0$, accurate to order k, while for the imaginary parts expressions, also linear in k, are taken, showing in a certain range, e.g., from $k = 0$ to $k = 0.05$, differences from the exact values, which are minimized by use of the method of least squares.

Author applies his procedure to complex k-values. In reviewer's opinion, one must be careful when considering stable oscillations since the derivation of Theodorsen's air forces imposes Im $k \leqslant 0$ and, indeed, the justification for this case given in appendix seems disputable to reviewer. Strip analysis is recom-

mended to obtain derivatives for a finite wing or tail, while replacing 2π by the slope of the lift curve in the circulatory air forces.

Final conclusion is that proposed method deserves recommendation since it will improve accuracy without appreciable increase of labor. A. I. van de Vooren, Holland

1707. Neumark, S., Longitudinal stability, speed and height, *Aircr. Engng.* **22,** 261, 323–334, Nov. 1950.

Paper investigates the effects of small variations in atmospheric density and speed of sound on dynamic longitudinal stability in level flight. The variations are caused by changes in height accompanying longitudinal disturbances. Results show a significant reduction in the period of the phugoid motion at high subsonic speeds and at supersonic speeds. Investigation included, necessarily, the effects of compressibility.

Arthur L. Jones, USA

1708. Scher, Stanley H., and Draper, John W., The effects of stability of spin-recovery tail parachutes on the behavior of airplanes in gliding flight and in spins, *Nat. adv. Comm. Aero. tech. Note* 2098, 67 pp., May 1950.

Tests on airplane models in the Langley free-flight and 20-ft free-spin tunnels show that unstable tail parachutes, when opened from gliding flight, resulted in extreme pitching and yawing oscillations which made sustained flight impossible. Stable tail parachutes used in the same test increased the stability of the model and sustained flight was made with ease. In both cases the parachute size was that normally required for spin recovery. Spin recovery was effective with either stable or unstable parachutes. Hemispherical parachutes with projected diameter equal to two thirds the laid-out-flat diameter of an unstable flat parachute gave equally good spin recoveries. Length of tow line did not materially change gyrations of an unstable tail chute. Increasing fabric porosity materially increased stability and decreased drag. H. P. Liepman, USA

1709. Berman, Theodore, Comparison of model and full-scale spin test results for 60 airplane designs, *Nat. adv. Comm. Aero. tech. Note* 2134, 15 pp., July 1950.

Model tests satisfactorily predicted full-scale recovery about 90% of the time. The remaining 10% were inconclusive but had value in predicting some details of the full-scale spins. Full-scale airplanes usually spin at angles of attack closer to 45° than corresponding models. Models generally spun with a lower altitude loss per revolution although high rates of descent were usually associated with smaller angles of attack, whether of model or full-scale airplane. Rates of rotation of the model were generally higher than those of the airplane for tail-damping ratios greater than 0.02 and vice versa. Downward tilt of the inner wing was usually greater on the airplane than on the corresponding model. Model tests resulted in conservative predictions of emergency-parachute sizes. H. P. Liepman, USA

1710. Lovell, Powell M., Jr., and Stassi, Paul P., A comparison of the lateral controllability with flap and plug ailerons on a sweptback-wing model, *Nat. adv. Comm. Aero. tech. Note* 2089, 19 pp., May 1950.

Free-flight tunnel tests for lift coefficients from 0.6 to the stall were made to obtain comparative data on the lateral control characteristics of the two types of control. The wing was of aspect ratio 3, taper ratio 0.5, and 38 degrees of sweepback. In addition, static force tests, and single-degree-of-freedom roll tests to check lag were made of the two installations.

The step ailerons consisted of 6 segments, each 0.1 of the semi-span with the center of each segment on the 70% chord line and perpendicular to the plane of symmetry. The maximum extension of the plug ailerons was 0.06 of the local chord.

For low speed control the plug ailerons were superior to the flap ailerons because the adverse yaw characteristics of the flap ailerons were more objectionable than the lag in the effectiveness of the plug ailerons. At the lowest lift coefficient tested, 0.6, the opposite conclusion was reached so that we can expect the flap aileron to be superior in the high speed range. From a practical standpoint it would seem that some combination of the two types of ailerons would give satisfactory lateral control throughout the speed range, not considering Mach number effects.

At the stall the model required flap ailerons in combination with the rudder for satisfactory control, or plug ailerons only.

Conrad A. Lau, USA

1711. Miller, R. H., A method for improving the inherent stability and control characteristics of helicopters, *J. aero. Sci.* **17,** 6, 363–374, June 1950.

Paper investigates the helicopter control and stability problem to establish whether satisfactory inherent characteristics may be achieved without major design changes and without use of automatic control devices. It is a continuation of author's study to achieve satisfactory stability and control characteristics of helicopter by overbalancing the rotor blades about the feathering axis and incorporating spring and viscous restraint in the control system. Miller shows that it is possible to achieve a degree of stabilization in pitch different from that in roll by applying dampening to nonrotating elements of the control system.

Discussion and theoretical treatment of subject is based upon helicopters having dual-rotating rotors to permit a simplification of required mathematics. Equations of motion for hovering flight are established and appropriate expressions for adapting these equations to forward flight are given. The section dealing with theoretical treatment would be of more use had the list of nomenclature been included in lieu of the reference to notation of a previous paper by author. The analysis does provide a number of variables so that designers can obtain desired control response characteristics for most flight conditions. In words of author, "It is not presented as a final solution to the question of helicopter stability and control and, at best, is simply a theoretical indication of what might be a promising field of flight investigation."

R. A. Young, USA

1712. Hohenemser, Kurt, A type of lifting rotor with inherent stability, *J. aero. Sci.* **17,** 9, 555–564, Sept. 1950.

It is shown that the inherent longitudinal static instability of a conventional helicopter with fixed-pitch rotor can be eliminated by means of an inclined (δ_3) flapping hinge-axis, by introducing a kinematic relation between the collective blade pitch angle θ and the coning angle a_0. This relation is of the form $\theta = C - Da_0$, where C and D are positive constants. Thus θ diminishes as a_0 increases. Theoretical calculations show that not only is positive static stability thus achieved, but also that partial blade stall during pull-ups or upward gusts can be avoided, because of lower values of thrust coefficients obtained. Moreover, it is also shown that pitch-cone change increases the velocity stability of the rotor, while it does not impair the damping in pitch of the helicopter. Morris Morduchow, USA

1713. Gessow, Alfred, An analysis of the autorotative performance of a helicopter powered by rotor-tip jet units, *Nat. adv. Comm. Aero. tech. Note* 2154, 27 pp., July 1950.

The autorotative velocity of descent of an assumed helicopter with rotor-blade-tip jet units is calculated. Because of the drag

of these units, absorbing a large amount of profile-drag power, the autorotative descent velocity is considerably higher than that of conventionally powered helicopters. This effect is shown to lessen at lower tip speeds; however, the rotor kinetic energy and the stall margin available for the landing maneuver are then reduced. It is pointed out that power-off descent velocities of pulsejet helicopters should be less than those of ramjet helicopters, since the pulsejets at present have higher ratios of net power-on thrust to power-off drag than the ramjets. In view of the dangerous increase of autorotative descent velocities due to inoperative jet units, author recommends that steps be taken to reduce the power-off drag of these units and to determine the maximum autorotative rate of descent that is acceptable to the pilot in steady gliding flight. Morris Morduchow, USA

1714. **Bisplinghoff, R. L., Isakson, G., and O'Brien, T. F., Gust loads on rigid airplanes with pitching neglected,** *J. aero. Sci.* **18, 1, 33–42, Jan. 1951.**

Paper contains an analysis of dynamic airplane response to a gust with an initial linear gradient. Results are presented in terms of a gust-gradient parameter and an airplane mass parameter for both incompressible and supersonic flow, the inadequacy of available solutions for high subsonic flow being pointed out. The incompressible flow analysis is based on the indicial responses developed by Wagner and Küssner, used in conjunction with the lift curve slope for the finite wing. Harvard Lomax, USA

1715. **Shuffelbarger, C. C., and Mickleboro, Harry C., Flight investigation of the effect of transient wing response on measured accelerations of a modern transport airplane in rough air,** *Nat. adv. Comm. Aero. tech. Note* **2150**, 18 pp., Aug. 1950.

Results of a flight investigation on a modern transport airplane to determine the transient effect of wing flexibility in gusts show that the measured peak acceleration increments at the center of gravity are, on the average, more than 20% higher than the true airplane acceleration increments. A slight change of acceleration discrepancy with change of wing weight and speed was indicated but could not be substantiated.

From authors' summary by P. Donely, USA

1716. **Walker, Harold J., and Ballantyne, Mary B., Pressure distribution and damping in steady roll at supersonic Mach numbers of flat swept-back wings with subsonic edges,** *Nat. adv. Comm. Aero. tech. Note* 2047, 57 pp., Mar. 1950.

A method based on linearized potential theory is presented for calculating pressure distribution and damping in steady roll at supersonic Mach numbers of flat sweptback wings having all edges straight and subsonic. A method is developed for streamwise wing tips but is applicable to cases having negatively raked tips. Illustrative examples included are calculations of pressure distribution and damping derivative of an untapered sweptback wing, and the damping derivative of a tapered sweptback wing. A comparison of these results with those from simple strip theory shows that strip theory is considerably in error in predicting pressure distribution but gives good approximation of the damping derivative. H. J. Allen, USA

1717. **Falkner, V. M., Sweepback and wing taper. The effect of sweepback on the lift, drag, and aerodynamic centre of a tapered wing,** *Aircr. Engng.* **22**, 260, 296–300, Oct. 1950.

Results are given of a series of calculations of the aerodynamic loading, forces, and moments for a family of four representative tapered wings of varying sweep in incompressible, nonviscous flow. Wings have an aspect ratio of 5.82, taper ratio of 0.32, and sweep angle varying from 20° sweepforward to 45° sweepback measured on the quarter-chord line. Both flat and twisted wings are included. Work was carried out using the vortex lattice method developed in earlier papers by author. In the present computations either an 84 or 126 lattice was used. Results are clearly presented in graphical and tabular form.

John R. Spreiter, USA

1718. **Margolis, Kenneth, Theoretical calculations of the lateral force and yawing moment due to rolling at supersonic speeds for sweptback tapered wings with streamwise tips. Subsonic leading edges,** *Nat. adv. Comm. Aero. tech. Note* 2122, 33 pp., June 1950.

Lateral force and yawing moment in a principal-body axis system due to combined incidence and roll are computed for a family of flat wings described in title. Method is based on linearized theory and uses the results of a previous application of Evvard's method for the approximate potential of the region inside the tip cone and the known results for a triangular wing for remaining region; from these the leading edge and tip suction forces are found. Graphs given cover a range of Mach numbers and shapes, including subsonic trailing edges for which the theory is not applicable; author states that use of the results for subsonic trailing edges is for rough estimates only. A simple transformation of sufficient accuracy is given for conversion to the stability axis system. J. M. Evans, Australia

1719. **Sechler, E. E., Williams, M. L., and Fung, Y. C., An initial approach to the over-all structural problems of swept wings under static loads,** *J. aero. Sci.* **17**, 10, 639–646, Oct. 1950.

Paper outlines in general terms the results obtained to date of a continuing investigation of the subject of stresses in thin swept wings. Authors review briefly results of theoretical, experimental, and electrical-analog solutions of the stresses and deflections of a thin cantilever plate of uniform thickness with varying angle of sweep. Loadings considered include uniformly distributed load, concentrated tip load, and torsional moment at tip. Future work contemplated under the program is indicated. Detailed results of this investigation may be found in *Air Force Technical Reports* AFTR-5761-1, -2, -3, and -4.

Robert M. Crane, USA

1720. **Margolis, Kenneth, Theoretical lift and damping in roll of thin sweptback tapered wings with raked-in and cross-stream wing tips at supersonic speeds. Subsonic leading edges,** *Nat. adv. Comm. Aero. tech. Note* 2048, 37 pp. Mar. 1950.

Paper presents design charts permitting estimation of the lift curve slopes and damping-in-roll derivatives for the class of wings indicated in title. John W. Miles, USA

1721. **Bird, John D., Some theoretical low-speed span loading characteristics of swept wings in roll and sideslip,** *Nat. adv. Comm. Aero. Rep.* 969, 15 pp., 1950.

See AMR 2, Rev. 780.

1722. **Lomax, Harvard, Sluder, Loma, and Heaslet, Max A., The calculation of downwash behind supersonic wings with an application to triangular plan forms,** *Nat. adv. Comm. Aero. Rep.* 957, 19 pp., 1950.

A method is developed, consistent with the assumptions of small perturbation theory, which provides a means of determining the downwash behind a wing in supersonic flow for a known load distribution. Analysis is based upon use of supersonic doublets which are distributed over the plan form and wake of the wing in a manner determined from the wing loading.

Equivalence in subsonic and supersonic flow of the downwash at infinity corresponding to a given load distribution is proved. In order to introduce the manipulative techniques which are subsequently employed, the unswept wing of infinite span is treated for supersonic speeds. Principal application in this report, however, is concerned with the downwash behind a triangular wing with leading edges swept back of the Mach cone from the vertex. Complete solutions are given for the chord plane in the extended vortex wake of the wing and for the vertical plane of symmetry. An approximate solution is also provided for points in the vicinity of the center line of the wake. From authors' summary

1723. Ribner, H. S., Time-dependent downwash at the tail and the pitching moment due to normal acceleration at supersonic speeds, *Nat. adv. Comm. Aero. tech. Note* 2042, 17 pp., Feb. 1950.

Time-dependent downwash behind a wing in a supersonic stream is analyzed for case when the angle of attack varies linearly with time. Result is applied to calculation of the contribution of the horizontal tail to the pitching moment and lift due to normal acceleration of airplane. Method employs an extension of an unpublished solution of the linearized potential equation for unsteady flow by Clifford S. Gardner. Pitching moment due to normal acceleration, together with damping in pitch, determines the damping of the short-period mode of longitudinal oscillation for an airplane.

From author's summary by M. D. Friedman, USA

1724. Mirels, H., and Haefeli, R. C., The calculation of supersonic downwash using line vortex theory, *J. aero. Sci.* **17,** 1, 13–21, Jan. 1950.

See AMR 3, Rev. 2404.

1725. Diederich, Franklin W., Approximate aerodynamic influence coefficients for wings of arbitrary plan form in subsonic flow, *Nat. adv. Comm. Aero. tech. Note* 2092, 17 pp., July 1950.

Using empirical results [see *NACA R.M.* L7107, 1948; not available to reviewer], a (matrix) procedure is derived for calculation of spanwise lift distributions. Modifications are given (1) if a wing with fuselage is considered; (2) if aeroelastic effects are taken into account. Author makes it plausible that fairly reliable results may be expected for the aeroelastic effect on symmetric spanwise loading, and that the results for basic loading are poorer than those for additional loading. A. van Heemert, Holland

1726. Bolz, Ray E., and Nicolaides, John D., A method of determining some aerodynamic coefficients from supersonic free-flight tests of a rolling missile, *J. aero. Sci.* **17,** 10, 609–621, Oct. 1950.

A technique is described for measuring rolling moments by observations on small test vehicles fired from a special gun in a free-flight ballistic range. Results for damping in roll and rolling moment due to offset rectangular fins at supersonic speeds are compared with linearized theory. Discrepancies (7–22%) are attributed to the airfoil section, a single wedge with blunt trailing edge. Principal conclusion is that the technique is a useful one.

W. J. Strang, Australia

1727. Tucker, Warren A., and Piland, Robert O., Estimation of the damping in roll of supersonic-leading-edge wing-body combinations, *Nat. adv. Comm. Aero. tech. Note* 2151, 20 pp., July 1950.

Report presents a linear-theory method for estimating C_{l_p}. The boundary conditions used for finding the loading on, for example, the right wing panel, are $w = py$ for $y > a$, and $w = -py\,(1 - 2a/h)$ for $y < a$, where y is distance measured along the span, a body radius, h distance parallel to the y axis from body center to the Mach line from the juncture of the body and wing leading edge, p rate of roll, and w vertical induced velocity. These conditions are correct when a/h equals 0 or 1. For $0 < a/h < 1$ the error introduced by the approximation is shown to be small. Values of C_{l_p} are given for configurations having rectangular and triangular wings; for most of the range of variables, net effect of body is to decrease damping in roll. Harvard Lomax, USA

1728. Bouten, Innes, Lift-curve slope for swept and unswept wings, *J. aero. Sci.* **17,** 3, 185, Mar. 1950.

A simple empirical formula is presented which is claimed to correlate satisfactorily with over 90 wings ranging in aspect ratio from less than 1 to over 12, more than half of which have sweepback, some as much as 60°. John R. Spreiter, USA

1729. Kainer, Julian H., and Marte, Jack E., Theoretical supersonic characteristics of inboard trailing-edge flaps having arbitrary sweep and taper Mach lines behind flap leading and trailing edges, *Nat. adv. Comm. Aero. tech. Note* 2205, 39 pp., Oct. 1950.

Generalized expressions in closed form have been obtained by means of linearized theory for the aerodynamic characteristics (lift-, rolling-moment-, pitching-moment-, and hinge-moment-coefficient derivatives) due to deflection of inboard trailing-edge flaps. The analysis considers the effects of Mach number and flap aspect ratio, taper ratio and sweep for the conditions where the Mach lines lie behind the flap leading and trailing edges. Flap configurations analyzed are limited to case of streamwise tips where the foremost Mach lines from the flap tips do not intersect root and tip chords of wing. Expressions for hinge-moment-coefficient derivative are limited to configurations where the flap-root-chord Mach line does not intersect the flap tip chord.

Design charts are presented for the rapid estimation of the characteristics due to flap deflection for control surfaces for which Mach lines from flap tips do not intersect on control surface. Some illustrative variations of characteristics with leading-edge sweep, aspect ratio, taper ratio, and Mach number are also presented.

From authors' summary by Herbert R. Lawrence, USA

1730. Beane, Beverly, The characteristics of supersonic wings having biconvex sections, *J. aero. Sci.* **18,** 1, 7–20, Jan. 1951.

To calculate the wave drags of wings with straight supersonic leading and trailing edges, method of source distributions was used. The biconvex profile was approximated as a $2N$-sided polygon; this permitted use of existing results for constant-slope wings, by superposition. The formulas for this method are given in some detail. An investigation indicated that N need not be taken greater than 16; consequently, this approximation was employed. A considerable range of sweepbacks, taper ratios, and aspect ratios has been covered, while Mach number and thickness ratio remain arbitrary. Comparison of biconvex and double-wedge wings indicates that the former are superior when their thicknesses are chosen for equal structural strengths. On the basis of limited comparisons with experiment, it is concluded that the linearized theory gives a reasonable first approximation for wings of this type. W. R. Sears, USA

1731. Cohen, Doris, Formulas and charts for the supersonic lift and drag of flat swept-back wings with interacting leading and trailing edges, *Nat. adv. Comm. Aero. tech. Note* 2093, 36 pp., May 1950.

Problem is considered of a wing with rectilinear plan form swept so that both leading and trailing edges lie within their respective

Mach cones; moreover, the Mach lines from the trailing-edge apex intersect the leading edge. Formulas and design charts are presented for lift in such a case, based on approximate formulas for lift distribution developed in *NACA T.N.* 1991, 1949. Charts cover a practical range of aspect ratios and plan forms of moderate taper, with tips parallel to stream. Leading-edge thrust and drag due to lift are also readily calculated from material presented. Numerical results and an application of the charts are included. From author's summary by G. H. Lean, England

1732. **Jacobs, Willy, Lift and moment changes due to the fuselage for a yawed aeroplane with unswept and swept wings,** *Flygtekn. Forsoksanst. Aero. Res. Inst. Sweden Rep.* 34, 51 pp., 1950.

First part discusses in simple terms the changes in incidence along the wing induced by fuselage when yawed, and shows that resulting change in lift distribution is skew-symmetric for an unswept wing, but is not so when wing is swept. In latter case, a pitching moment results, which, although proportional to the square of the angle of side slip, can be readily appreciable. Relatively simple methods of estimating these effects are indicated. Second part discusses results obtained from wind-tunnel tests of wing-fuselage combinations, involving wings of aspect ratio 4.5, sweepback angles of 0° and 40°, and various wing positions. Detailed results are given of spanwise lift distributions, rolling moments and pitching moments for various angles of incidence and yaw. The agreement with theory is satisfactory.

A. D. Young, England

1733. **Sivells, James C., and Spooner, Stanley H., Investigation in the Langley 19-foot pressure tunnel of two wings of NACA 65-210 and 64-210 airfoil sections with various type flaps,** *Nat. adv. Comm. Aero. Rep.* 942, 23 pp., 1949.

Need for information on the use of flaps as high-lift devices for aircraft having relatively thin wings (normally with low $C_{L_{max}}$) prompted this investigation at high Reynolds numbers of typical wing forms with aspect ratio of 9 and taper ratio of 0.4. Split, single-slotted, and double-slotted flaps were tested with $C_{L_{max}}$ reaching 205% of flap-neutral values. Effects of leading-edge roughness and fuselage interference on flow distribution were investigated with tufts to indicate degree of local stalling on the wings. Measurements of lift, drag, and pitching moments are submitted in conventional nondimensional form, while stalling characteristics are illustrated by pictorial sketches of wing plan forms shaded to indicate flow characteristics. Although results obtained are limited to specific wing forms tested, and stall patterns, as pointed out by author, are unsymmetrical, owing to inconsistencies in the data when tested near maximum lift, enough information is given on wings tested to permit evaluation of trends and their application to other wings.

Robert S. Ross, USA

1734. **Flax, Alexander H., On a variational principle in lifting-line theory,** *J. aero. Sci.* **17**, 9, 596–597, Sept. 1950.

By replacing the Prandtl integral equation with the equivalent variational equation, author shows that a number of methods for determining lift distributions can be considered from one point of view. This becomes evident by expansion of the lift distribution in a series $c_l = \Sigma a_n f_n(x)$, resulting in a decomposition of the variational equation into an infinite number of equations in the a_n's. Using particular functions $f_n(x)$ the numerical methods of Lotz and Sears appear. The approximation method of Schrenk can also be related to the variational equation.

Author gives a physical interpretation of the variational equation and remarks that principle can be extended to more complicated cases. A new approximation method based on the variational principle is suggested, operating with relatively few terms and giving good results.

H. G. Loos, Holland

1735. **Sivells, James C., An improved approximate method for calculating lift distributions due to twist,** *Nat. adv. Comm. Aero. tech. Note* 2282, 35 pp., Jan. 1951.

Commonly used approximate methods of calculating spanwise lift distributions at subsonic speeds [*NACA T.M.* 948 by Schrenk, and *NACA R.M.* L7107 by reviewer] lack rational basis and are not very satisfactory for very high or low aspect ratios, nor for angle-of-attack distributions due to twist. On the basis of the lifting line result for the lift of a twisted wing obtained by Gdaliahu [*Aero. Res. Counc. Rep. Mem.* 2261] and of approximate lifting-surface and sweep corrections, present paper derives simple method for calculating approximate lift distributions due to twist in terms of the lift distribution for uniform angle of attack. Examples presented for various types of twist, and for various plan forms, show the lift distributions calculated by this method to be in excellent agreement with those calculated by more refined theoretical methods.

Franklin W. Diederich, USA

1736. **Adams, Gaynor, J., Theoretical damping in roll and rolling effectiveness of slender cruciform wings,** *Nat. adv. Comm. Aero. tech. Note* 2270, 30 pp., Jan. 1951.

Damping-in-roll and rolling moment due to differential incidence of opposite wing panels, using slender wing theory [*NACA Rep.* 835], allowing for interference between fin and wings for a cruciform shape. Analysis uses conformal transformation to map the outside of a circle on region outside a symmetrical cross. Damping in roll for cruciform wing is 62% greater than that of plane wing having same aspect ratio. Rolling effectiveness of cruciform wing with four deflected panels is 6% less than that of plane wing.

A. W. Babister, England

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 1630, 1706, 1725)

1737. **Rott, Nikolaus, Forms of wing oscillations in a plane compressible potential flow** (in German), *Z. angew. Math. Phys.* 1, 6, 380–410, Nov. 1950.

Mechanism of two-degree flutter is discussed. Based on energy considerations for two-dimensional, subsonic, sonic, and supersonic flow, bending-torsion complex amplitude ratios and stability loci are determined in terms of reduced frequency and Mach number. Conclusions are drawn for avoidance of flutter. Special attention is paid to sonic case with analytical and numerical values of flutter derivatives given. As author notes, sonic results are not applicable to three-dimensional plan forms (since "strip" theory is void due to dominant effects of wing edges). Reviewer notes that author's quasistationary analysis is incorrect (due to neglect of first-order frequency terms in pressure and continuity equations) and that results are useful primarily in presenting picture of flutter and not for practical flutter analysis, where determination of actual reduced frequency is tantamount to solving flutter problem.

John W. Miles, USA

1738. **Winson, Jonathan, The solution of aeroelastic problems by electronic analogue computation,** *J. aero. Sci.* **17**, 7, 385–395, July 1950.

Aeroelastic equations of motion are developed for coupled bending and torsion of a wing. The "lift deficiency" function is approximated by a double negative exponential. Following a general discussion of the principles of an analog computor of the REAC type, the operational scheme for setting up the equations

of motion in the computer is described, including a special RC network to represent the lift deficiency terms. Separate machine runs are made for each of several forward velocities, and oscillograph records are obtained which represent wing oscillations as a function of time. Oscillations diverge above the flutter speed. Data are given for two different wing systems. Flutter speeds obtained by the analog solution agree closely with results obtained by numerical calculations. Dana Young, USA

1739. Watkins, Charles E., Effect of aspect ratio on the air forces and moments of harmonically oscillating thin rectangular wings in supersonic potential flow, *Nat. adv. Comm. Aero. tech. Note* 2064, 52 pp., Apr. 1950.

Formulas are derived for spanwise distribution of lift and pitching moment as well as total lift and moment. Motions considered are vertical translation and pitching about an arbitrary spanwise axis. Results are based upon a finite series for velocity potential including third power of oscillation frequency. Governing differential equation is satisfied to highest power of frequency considered, and linearized boundary conditions are satisfied exactly. Aspect ratio and Mach number are limited by assumption that Mach lines from one tip do not intersect opposite wing tip. Effects of variations in aspect ratio and Mach number are shown graphically for one location of pitching axis and one value of reduced frequency. M. J. Turner, USA

1740. Grossman, E. P., Flutter (translation from Russian), *Hdqtrs. Air Mat. Comm. Dayton, O. Transl.* no. F-TS-1225-IA, 393 pp., May 1949.

A translation of one of the few available texts dealing in some detail with the "potential flow" flutter problem. However, save as an interesting and refreshing discussion of the dynamic properties of flutter systems, it does not meet the objective of supplying the information required as a basis for present-day flutter analyses. Chief reason for this shortcoming is that the aerodynamic treatment throughout is from the pseudostatic viewpoint. By thus simplifying the aerodynamics, author aims toward a concise treatment of the problem, within the scope of the practicing design engineer, and amenable to the establishment of relatively simple design criteria. Today, it is, of course, realized that this approach is unsatisfactory, since acceptable accuracy in flutter analyses can only be achieved through incorporation of quite accurate unsteady aerodynamic theories in the work.

The bending-torsion flutter of a cantilever wing is given considerable attention, including derivation of the flutter condition directly from the differential equations for wing bending and twisting. The dynamic character of flutter systems is thus made clear. The use of generalized coordinates is also discussed, although in a cursory and unsatisfactory fashion. Design considerations for making wings flutter-free are given considerable attention. Finally, the flutter of wings with aileron participation, and flutter of tailplane systems is given brief attention.

Although no publication date for the original text is given, the references do not extend beyond 1936. For publication around that time, the text represents a substantial contribution to the art. For current purposes, however, its scope and methods are severely outdated. M. Goland, USA

1741. Miles, John W., A formulation of the aeroelastic problem for a swept wing, *J. aero. Sci.* 16, 477–490, Aug. 1949.

Structural theory of wings having their elastic axes inclined relative to the axis of symmetry of the airplane is dealt with in this paper on the basis of some plausible simplifying premises, as follows: Aerodynamic centers and elastic axes lie on parallel straight lines. Cross sections originally perpendicular to the elastic axis remain parallel to one another. Chordwise deformation is neglected. Method requires that angle of incidence be measured for sections parallel to the airplane axis of symmetry, and twisting angle for sections perpendicular to the elastic axis. A contradiction appears when the change of angle of incidence is expressed in terms of the angle of twist, the bending angle (slope) along the elastic axis, and the sweep angle. This is also neglected.

Wing is treated like a beam, the centroid line coinciding with the elastic axis, so that the differential equations of elementary theory can be applied. Aerodynamic forces and their twisting moments are expressed by lift- and moment-coefficients, with consideration of the influence of the deflections on the angles of incidence. Induction due to trailing vortex sheet of lifting-line theory, and preferably explained by lifting-surface theory, is also discussed. A source is mentioned in connection with the lift distribution. Many readers will not see where this single source is located and how it can determine an entire lift distribution. Inertia forces and moments due to accelerations are given in regular form.

From the differential equations, energy equations are derived by means of multiplication by the deflection functions and then by integration in parts over the length of the elastic axis. Boundary conditions of the differential and energy equations at the wing root seem to reviewer insufficient in regard to the bending and twisting fixation of the root section which is not perpendicular to elastic axis.

After a further discussion of the influence of control surface displacements, different possible methods of solution of the differential and energy equations are indicated. One more explicit approximate integration is given and is also applied for determination of the divergence speed. H. J. Reissner, USA

1742. Scruton, C., Experiments on tail flutter, *Aero. Res. Counc. Lond. Rep. Mem.* 2323, 137 pp., 1949.

Report describes a general experimental investigation of tail flutter in which systematic tests of flutter characteristics of models of both twin and single fin-rudder tail units were made. From author's summary

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 1446, 1512, 1573, 1711, 1772, 1781, 1824, 1825)

1743. Falkner, J. C., Napier, D. W., and Kellstedt, C. W., Latest technique for quick starts on large turbines and boilers, *Trans. Amer. Soc. mech. Engrs.* 72, 8, 1111–1136, Nov. 1950.

The first quick starts made on the 1200-psi 900-F topping turbines at the Waterside Station were described in a paper presented in June, 1947. Purpose of present paper is to review the high lights of the earlier presentation and to relate developments since then. From authors' summary

1744. Perycz, Stefan, Principles of geometrical design of turbine nozzles (in Polish), *Przegl. mech.*, no. 4–6, 8 pp., 1950.

Author is concerned primarily with geometrical design of diaphragms with cast-in guide blades. The considerations apply, however, to any other type of turbine nozzles. Author throws away cylindrical sections developed in a plane as they do not offer any particular value or advantage in design and production practice. Following pure geometrical reasoning with a number of simplifying assumptions, author discusses two problems: 1 Proper shape of the nozzle axis in the longitudinal section, and 2 determination of the real vane angle at outlet. Special con-

siderations are devoted to proper choice of boundary walls at nozzle outlet. Theoretical investigations are followed up by some examples of detailed geometrical design.

Reviewer believes that this paper represents the first endeavor for a more detailed explanation of the problem than usually appears in turbine literature, and that paper is, therefore, of considerable value to engineers designing steam turbines.

R. Szewalski, Poland

1745. **Szewalski, Robert, A new theory of labyrinth-packings** (in Polish), *Przegl. mech.*, no. 4–6, 8 pp., 1950.

The theory of Stodola is based on a number of assumptions. His formulas for calculating labyrinth packings differ, therefore, from an exact solution now presented by author. The new theory and calculation method is basically similar to that presented by C. S. L. Robinson [*J. appl. Mech.*, Dec. 1948]. Author claims his priority (1942), and that he has been lecturing on the new theory since 1944. Paper contains a number of original diagrams and numerical examples.

The ideal case of a perfect labyrinth packing (i — const) is followed up in discussion by the more general representation of the problem with ζ, the coefficient of carryover velocity, as an additional parameter.

K. Zarankiewicz, Poland

1746. **Kochendorfer, Fred D., and Nettles, J. Cary, An analytical method of estimating turbine performance,** *Nat. adv. Comm. Aero. Rep.* 930, 13 pp. 1949.

Analysis is based on one-dimensional flow through channels formed by the cascade profiles at the pitch diameter. Direction of relative flow leaving the cascades is considered independent from angle of attack and Mach number for subsonic discharge; for supersonic discharge, deviation from this direction is calculated as Prandtl-Meyer expansion. Total pressure loss through cascade is accounted for by experimentally determined blading loss parameters. With this method, it is possible to calculate performance trends of a gas turbine over a wide range of operating conditions.

Andrew Fejer, USA

1747. **Martinuzzi, P. F., Continental and American gas-turbine and compressor calculation methods compared,** *Trans. Amer. Soc. mech. Engrs.* **71**, 325–333, May 1949.

European methods for calculation of axial compressor bladings are outlined. Dimensionless velocity triangles are used and the cascade characteristics are presented in terms of pressure coefficient, flow coefficient, profile glide angles, and degree of reaction. Examples of blading types, presented in terms of the velocity triangle at the pitch diameter include stages with axial discharge from rotor, symmetrical stages, stages with axial rotor inlet, and stages with symmetrical stators. Axial velocity is always selected constant at all diameters, and radial equilibrium is insured before and after the rotor by the use of constant circulation along the blades and free vortex flow in the gaps between cascades. Relative merits of above blading types and their application to gas turbine compressors are also presented.

Andrew Fejer, USA

1748. **Stanitz, John D., and Ellis, Gaylord O., Two-dimensional compressible flow in centrifugal compressors with straight blades,** *Nat. adv. Comm. Aero. tech. Note* 1932, 80 pp., Aug. 1949.

Paper presents an application of an earlier one by J. D. Stanitz [see AMR **2**, Rev. 375]. Six numerical examples are calculated for steady, compressible, nonviscous flow in centrifugal compressors with straight blades, in which the center line of the passage generates a right circular cone about axis of the runner. A seventh example is presented for incompressible flow. Numerical values of parameters are selected in order to show effects of variations in each separately, i.e., flow rate, impeller-tip Mach number, exponent for variation of passage height ratio with radius ratio, number of blades. Numerical results are presented in plots of streamlines, constant Mach number lines, and constant pressure-ratio lines. Correlation equations are developed whereby flow conditions within any impeller with straight blades can be determined from the flow conditions of standard solution presented.

Solutions are limited to a region of the compressor, including impeller tip, that was considered to be unaffected by the inlet configuration of the impeller and by the diffuser vanes. Many interesting conclusions result from this work in addition to that quoted in above-mentioned review; for example, various means to avoid formation of a wheel-type eddy on the driving face of the blades.

Gino Moretti, Argentina

1749. **Hamrick, Joseph T., Ginsburg, Ambrose, and Osborn, Walter M., Method of analysis for compressible flow through mixed-flow centrifugal impellers of arbitrary design,** *Nat. adv. Comm. Aero. tech. Note* 2165, 29 pp. Aug. 1950.

A method is presented for analysis of the compressible flow between the hub and shroud of mixed-flow impellers of arbitrary design. Axial symmetry is assumed, but the forces in the hub to shroud plane, which are derived from tangential pressure gradients, are taken into account. Method of calculation is based on one proposed by Flügel, for analysis of flow in the blade-to-blade direction and discussed by Stodola.

Method is applied to an experimental mixed-flow impeller. The analysis of flow showed that the rotational forces, blade curvature, and hub-shroud profile can introduce severe velocity gradients along the hub and shroud surfaces. Choked flow in the impeller inlet as determined by the analysis was verified by experimental results. Method is limited to channel flow and therefore to impellers of high solidity, and the assumption for the basic equations loses its validity in vicinity of the impeller inlet and exit. The calculated result refers to the mean velocity from blade to blade. Considering complexity of the problem, the analysis shows remarkable agreement with test data and indicates the great value of the analytical investigation. It must be pointed out that the results, within the above-mentioned limitations, include both compressible and three-dimensional flow. Presented method is most important for analysis of the inlet conditions of mixed flow impellers.

H. E. Sheets, USA

1750. **Witty, R., Characteristics of diffusion pumps,** *Brit. J. appl. Phys.* **1**, 9, 232–237, Sept. 1950.

The essential characteristics of a diffusion pump are discussed, and apparatus and methods for determining them are described. Comparison of the performance of different pumps is discussed, and, as an example, experimental results are given for relation between performance and gap width. Finally, theories of diffusion-pump operation are discussed in the light of these experimental results and some future lines of experiment indicated.

From author's summary

1751. **Lieblein, Seymour, Turning-angle design rules for constant-thickness circular-arc inlet guide vanes in axial annular flow,** *Nat. adv. Comm. Aero. tech. Note* 2179, 23 pp., Sept. 1950.

A survey of data from investigations of axial-flow compressor-inlet guide vanes with circular-arc, constant-thickness sections and axial air inlet was conducted to establish a relation between vane camber and air-turning angle for use in the design of this type of vane. Two design rules were obtained for vane set at

zero angle of incidence, solidities from 1.4 to 1.7, and inlet Mach numbers of approximately 0.3. From author's summary

1752. Casacci, S. X., and Jarriand, P., The measurement of hydraulic thrust on a vertical Francis turbine (in French), *Houille blanche* **5**, 3, 326–332, May-June 1950.

Method used by authors consists of measuring deformation of girder supporting the thrust bearings; the dynamometer formed by this girder is calibrated in terms of weight of the moving parts. Authors give test results for three turbines. From these tests they also deduce the coefficient in a semi-empirical formula enabling determination, as a function of specific speed, of maximum hydraulic thrust of a Francis turbine of known construction, not operating under load.

Translation from authors' summary

1753. Spannhake, E. W., Hydrodynamics of hydraulic torque converter, *Trans. Soc. auto. Engrs.* **3**, 4, 592–608, Oct. 1949.

Analysis explains basic behavior of torque converters by the application of simple hydrodynamic theory. The equations make possible the evaluation of the relative merit of torque converters having various performance characteristics. The equations are useful also for indicating the influence of design changes.

From author's summary

1754. Schmitt, Heinz, E., Turbojet afterburning without an afterburner, *Aero. Engng. Rev.* **9**, 12, 18–24, Dec. 1950.

To avoid any drag-inducing installations in the tailpipe and to achieve a stable combustion in spite of the high velocity of exhaust gases, fuel is injected before the turbine nozzles, turbine wheel being used as a flameholder. Because of natural ignition delay, fuel does not burn until it has passed through turbine wheel. It is concluded that, if diffuser section and burner section are united as one, simplicity and high efficiency with a minimum of space and weight requirements can be attained.

From author's summary

1755. Doumerg, René, Theory of operation of a turbo-jet with simple flow (in French), *C. R. Acad. Sci. Paris* **231**, 5, 329–331, July 1950.

Author, using two graphs, shows he can determine quickly the relation between pressure ratio in the compressor and over-all temperature rise in compressor and combustor. One of the graphs is obtained from the relation connecting mass flow with area of exhaust-nozzle throat and expansion ratio through nozzle; the second from the more involved relation between the same mass flow, throat area of the turbine nozzle, compressor pressure ratio, and compressor-combustor temperature ratio. Velocity of flight can be easily taken into account if the graphs are plotted in logarithmic scale. Other results that can be obtained from these graphs are also quoted. No numerical example is given, and no graphs are shown. Luigi Crocco, USA

1756. Poggi, L., Contribution to the study of pulse jets I, II (in Italian), *Aerotecnica* **29**, 5, 6; 288–296, 356–363; Nov., Dec. 1949.

Some particular solutions of the equation for a one-dimensional flow of a gas in a cylindrical tube are developed, showing how they can be combined for the solution of some elementary problems. Results are applied to the study of possible systems of pulse jets constituted essentally for a cylindrical tube provided with a check valve. A system is also considered in which there is a rear valve which remains closed during filling of the tube, so that the fluid in it undergoes a compression before the explosion. Partial carburation of the fluid is also considered. Results are presented in curves which give the obtainable pulse and the efficiency as a function of the degree of carburization of the air.

Translation from author's summary

1757. Dahl, Andrew I., and Fiock, Ernest F., Response characteristics of temperature-sensing elements for use in the control of jet engines, *J. Res. nat. Bur. Stands.* **45**, 4, 292–298, Oct. 1950.

Rate at which a temperature-sensing element located in gas stream of a jet engine responds to sudden changes in temperature is of great practical importance in control and operation of such an engine. Factors that determine rate of response are discussed, and significance of characteristic time is emphasized. It is shown that laboratory determinations of characteristic time must be made under simulated engine conditions in which rate of heat transfer by forced convection is the controlling factor. Apparatus used at the NBS for measuring characteristic times is described, and typical results presented. Rate of response of a device in a jet engine varies greatly with engine speed and with flight altitude, so that satisfactory performance of a temperature-actuated control system can be expected only if sensing element responds with sufficient rapidity under starting conditions and at flight ceiling. Reviewer's remark: Equation (6) should read: $-\log_e (T_2 - T) = t/\tau + \text{const}$; equation (8) should read: $d(\Delta T)/dt = (\Delta T_0/\tau)e^{-t/\tau}$.

From authors' summary by A. W. Jones, USA

1758. Manson, S. V., Regenerator-design study and its application to turbine-propeller engines, *Nat. adv. Comm. Aero. tech. Note* 2254, 52 pp., Jan. 1951.

A study is presented of regeneration as a means of improving the load-range performance of turbine-propeller aircraft. Efficacy of regeneration for turbine-propeller engines was evaluated by comparing the cargo capacity of a typical airplane powered by a regenerated engine with cargo capacity of an airplane powered by an unregenerated engine. In each case, the turbine-propeller engine has a compressor of optimum pressure ratio for the operating conditions investigated and the regenerated engine has the optimum regenerator core among the many designed for the study.

Results indicate that gains from regeneration are small except near sea level for flight distances of 3000 or more miles and at speeds of about 300 mph. R. C. Binder, USA

Flow and Flight Test Techniques

(See also Revs. 1622, 1685, 1726, 1733, 1757)

1759. Prescott, Rochelle, and Gayhart, E. L., A method of correction of astigmatism in schlieren systems, *J. aero. Sci.* **18**, 1, p. 69, Jan. 1951.

Schlieren system consisting of point light source and 7° off-axis z-arrangement of two $f/6$ paraboloid mirrors is corrected for astigmatism by insertion of plano-cylindrical lens between light source and first mirror. Wallace F. Davis, USA

1760. McLellan, Charles H., Williams, Thomas W., and Bertram, Mitchel H., Investigation of a two-step nozzle in the Langley 11-inch hypersonic tunnel, *Nat. adv. Comm. Aero. tech. Note* 2171, 71 pp., Sept. 1950.

Nozzles for high supersonic speeds require high ratios of cross-section area. Therefore, one-step nozzles of two-dimensional design have thin slitlike throats. The construction of such nozzles leads to difficulties which are avoided with two-step nozzles.

Authors tested a nozzle for a Mach number 6.98 which was designed by the method of characteristics. In a tunnel with a 10-in. × 10-in. test section, the throat area was 1.5-in. × 0.667-in., compared with 10-in. × 0.1-in. with a one-step nozzle. The first expansion was to produce $M = 4.36$, the second expansion with plane turned 90°, the design Mach number. Tunnel stagnation pressure was varied from 2 to 44 atm, stagnation temperature such that liquefaction of the air was avoided with maximum of 850 F. Nozzle flow and flow in the test section were tested by measurements of static pressure, total pressure and total temperature, disturbance flow patterns by schlieren photographs. Special low-absolute-pressure capsules and stagnation temperature probes were developed.

Results show a heavy boundary layer originating at the parallel walls of the first expansion. The effect of this layer can be tracked downstream into the test section. No appreciable core of undisturbed flow is left. Authors conclude from these facts that two-step nozzles have worse characteristics than conventional one-step nozzles.

Reviewer believes that tests are sufficient to draw this conclusion. There is, however, a surprisingly great decrease of total temperature in the boundary layer which needs explanation. There may be a pressure dependency of the temperature probe. Otherwise, the results are affected by heat losses through the walls or by inleaking cold air. Heinrich Ramm, USA

1761. **Kuntz, Jean, A bench for tests at transonic speeds** (in French), *Génie civ.* **126,** 21, 397–399, Nov. 1949.

About two thirds of the paper is devoted to elementary introduction and simple test methods. Interesting data are given about the suspension of the model (which slides on rails and is driven and stopped by rockets), the rail track, and the electromagnetic velocity measurements. A built-in balance registers the aerodynamic forces and moments acting upon the model, the influence of the accelerations of ± 70 g being compensated.

S. F. Erdmann, Holland

1762. **Harris, Orville R., Determination of the rate of roll of pilotless aircraft research models by means of polarized radio waves,** *Nat. adv. Comm. Aero. tech. Note* 2023, 22 pp., Feb. 1950.

A method is presented for determining the rate of roll of free-flight aerodynamic models by means of polarized radio waves. A discussion of the technique used and a description of equipment are included. Through use of this technique the direction of roll, as well as zero roll, is uniquely determined. Analysis of data obtained and data accuracy are discussed and examples are given to show how the record-reduction method affects the accuracy of the data. In conclusion, a method for increasing the accuracy and reducing work-up effort is discussed.

From author's summary

1763. **Gracey, William, Coletti, Donald E., and Russell, Walter R., Wind-tunnel investigation of a number of total-pressure tubes at high angles of attack. Supersonic speeds,** *Nat. adv. Comm. Aero. tech. Note* 2261, 48 pp., Jan. 1951.

Angle of attack of 20 total pressure tubes having cylindrical-, conical-, and ogival-nose sections was varied between −15° and 45° at Mach numbers 1.62, 1.94, and 2.40 (for results obtained with the same tubes at subsonic speeds see *NACA R.M.* L50G19). In all but a shielded type [*NACA T.M.* 775], the range of angle of attack over which the tubes recorded correctly was appreciably greater at supersonic speeds than at subsonic speeds. Results are presented without detailed analysis.

George Rudinger, USA

1764. **Morkovin, M. V., Design of a device for measurement of free-stream static pressure at supersonic speeds,** *Aero. Engng. Rev.* **9,** 12, 25–28, Dec. 1950.

An excellent review of the requirements for a device to measure free-stream static pressure at supersonic speeds. A double-wedge airfoil with a flat area, where the static pressure orifices are located outside the Mach cones, is recommended as a possible design which would give accurate values independent of angle of attack, angle of yaw, and Mach number.

If experimental results of a prototype confirm the predictions, this device should prove a useful instrument for supersonic-flow investigations. Irvine I. Glass, Canada

1765. **Scholz, N., Measurements of force and pressure distribution on wings with small aspect ratio** (in German), *Forsch. Geb. Ing.-Wes.* **16,** 3, 85–91, 1949/1950.

Force and pressure distribution were measured in a wind tunnel on four rectangular flat plates and four airfoils, in both cases of the aspect ratios 3, 2, 1, and 0.5. Results are compared with lifting-surface theory given by author.

From author's summary

1766. **Wuest, W., Bellows manometer** (in German), *Arch. tech. Messen,* no. 166, V1343-10, Nov. 1949.

1767. **Lovesey, A. C., Modern methods of testing aero-engines and power plants,** *J. Roy. aero. Soc.* **54,** 474, 327–358, June 1950.

Paper describes several techniques and instruments used in testing gas-turbine aircraft engines and components. Measurements include gas pressure, temperature and velocity, running clearances, and oscillations of operating turbine blades. Some test results are presented. Test program planning is discussed.

A. D. St. John, USA

1768. **Eubank, William R., Precision thermostat for high temperatures,** *Rev. sci. Instrum.* **21,** 10, 845–851, Oct. 1950.

A precision thermostat has been developed, based upon that of Roberts, for control of high temperatures (1000–1550 C). With the instrument described, temperatures are readily maintained constant within ±0.1 C for several hours, and within ±1.0 C for several days. In this method of control a thyratron is used in conjunction with an a-c bridge and high gain amplifier. The phase angle of the unbalanced bridge regulates firing of the thyratron. Control is highly sensitive, independent of normal fluctuations in line voltage, and reliable for periods of continuous operation from a few minutes to several days.

From author's summary

1769. **Nichols, Mark R., and Keith, Arvid L., Jr., Investigation of systematic group of NACA-1 series cowlings with and without spinners,** *Nat. adv. Comm. Aero. Rep.* 950, 100 pp., 1949.

An investigation has been conducted in the Langley propeller-research tunnel to study cowling-spinner combinations based on the NACA 1-series nose inlets and to obtain systematic design data for one family of approximately ellipsoidal spinners.

From authors' summary

1770. **Conlin, L. T., Comparison of results of tests in the NACA, RAE and NRC spinning tunnels,** *Nat. Res. Counc. Canad. Rep.,* no. Ma-221, 7 pp. Mar. 1950.

Results of model spinning tests made in the NRC spinning tunnel on a $^1/_{16}$th scale model Curtiss-Wright SC-1 aircraft were found to agree with results of previous model spinning tests made in the NACA spinning tunnel, and results of model spinning tests on a $^1/_{20}$th scale "Prentice" (prototype) aircraft agreed with re-

sults of previous tests made in the RAE spinning tunnel with the exception that the NRC tests showed a slightly higher value of the threshold of recovery" for the Prentice than did the RAE tests.

Agreement between model and full-scale tests was good in the case of the Curtiss-Wright SC-1 as both model and full-scale tests showed recovery characteristics to be unsatisfactory. However, the model spin tests on the Prentice indicated no flat spinning tendency and considerable margin in recovery, whereas in full-scale spinning trials it was found that the spin would become flat after a few turns and that normal manipulation of the controls would not bring about recovery from such a spin.

From author's summary

1771. Bleakney, Walker, White, D. R., and Griffith, W. C., Measurements of diffraction of shock waves and resulting loading of structures, *J. appl. Mech.* **17**, 4, 439–445, Dec. 1950.

Paper is concerned with the evolution in time of a flow pattern about an obstacle initially in still air, passed over by a sudden front of high-speed flow induced by a shock wave. Experiments were carried out in a shock tube of 4-in. × 18-in. cross section. Various models were mounted in the test section and observations were usually made by means of interferograms taken both in still air and in the disturbed flow. From these, contours of constant density were plotted. Absolute fringe shift was determined from shift of the zero or white light fringes. Pressure at any point of the flow field was obtained from the density by assuming isentropic flow behind the incident shock wave. Error thus introduced did not exceed a few per cent and was usually considerably smaller.

Models tested included various shapes of blocks and wedges, a semicylinder, an airfoil at a small angle of attack, and two rectangular blocks arranged behind each other. Plots of fringe shifts observed are presented for various times after the initial shock wave makes contact with the model. A few pressure distributions on surface of a model are also given. From such plots, the transient forces acting on a model can be obtained. Experiments of this type may also be used to find regions in the flow where instruments such as pressure gages may be mounted to record free flow variations.

George Rudinger, USA

Thermodynamics

(See also Revs. 1674, 1755, 1820)

1772. Mullen, James W., II, Fenn, John B., and Garmon, Roland C., Burners for supersonic ram-jets. Factors controlling over-all burner performance, *Indust. Engng. Chem.* **43**, 1, 195–211, Jan. 1951.

Influence upon efficiency and air specific impulse of air-fuel ratio, mixture quality, igniter type, inlet-air temperature, combustion-chamber length, and air mass flow was studied for a 2-in. burner simulating conditions encountered in a ramjet's sea-level flight at $M = 1.5$ to 2.0. Results are interpreted in terms of variation of fuel volatility, ignition times and temperatures, and flame velocities with heat release and flow characteristics within the burner.

B. L. Hicks, USA

1773. Clark, Thomas P., Method for determining distribution of luminous emitters in cone of laminar Bunsen flame, *Nat. adv. Comm. Aero. tech. Note* 2246, 28 pp., Jan. 1951.

Report shows how the distribution across the reaction zone may be determined for a given radiation emitter in a laminar Bunsen flame. For simplification, any portion of the conical flame treated is considered equivalent to a cylinder of the same radius. Method requires, first, establishing the change in intensity of the image of a luminous cylindrical shell, viewed normal to its axis [Hubner and Klaukens, *Ann. Phys.*, Folge 5, **39**, 33–50, 1941], and, second, applying this relation to case of a luminous cylinder comprising many concentric shells of differential thickness, each with a different concentration of the emitter, i.e., the case of the flame cross section having a variable distribution of emitters.

Selection of an appropriate photographic filter permitted an experimental demonstration of the method for the C_2 emitter in a propane-air Bunsen flame. The C_2 emitter zone was approximately 0.25 mm thick with a maximum emitter distribution near the center of the zone for all fuel air ratios. Method should be applicable to other emitters and other luminous reactions; its value would be to assist in analyses of steps in chemical reactions.

W. T. Olson, USA

1774. Hahnemann, H., and Ehret, L., Effect of intense sound waves on a stationary gas flame, *Nat. adv. Comm. Aero. tech. Memo.* 1271, 35 pp., July 1950 (translation from *Z. tech. Phys.* **24**, 10–12, 228–242, 1943).

The technically important effects of turbulence and of acoustical disturbances upon flame characteristics were studied by irradiating a flame seated on a Mache-Hebra nozzle with sound waves of as high as 12.5 watts/cm² intensity. Only small changes of flame speed could be distinguished, but when the acoustical particle velocity became much greater than the flow velocity the flame adjusted to a potential flow pattern rather than to jet flow. An electrolytic tank was used to predict the potential flow (cont.) for the nozzle.

B. L. Hicks, USA

1775. Tuck, L. Dallas, The thermodynamics of some non-isothermal systems, *J. chem. Phys.* **18**, 9, 1128–1134, Sept. 1950.

Article shows theoretically why the Soret equilibrium and the emf of a thermocell are not affected by the irreversible heat transfer associated with them. It thus validates the use of relationships which are known to be applicable but are theoretically justified only for isothermal equilibrium.

Jack D. Bush, USA

1776. Plank, R., and Riedel, L., A new criterion for the shape of the vapor-pressure curve at the critical point (in German), *Ing.-Arch.* **16**, 3–4, 255–266, 1948.

Authors examine the differential ratio $\alpha = d\ln p/d\ln T$ where p and T denote the saturation pressure, or vapor pressure, and the temperature on the Kelvin scale. The value of α was computed for 26 of the 30 substances whose vapor pressures were measured by Sidney Young, and also for 9 other substances measured at the Leiden Laboratory. The latter list included argon and neon, along with four diatomic gases. Finally, the case of water was given detailed consideration in view of the superior degree of accord in the measurements reported by independent observers. In all except two substances, α falls to a constant value in the vicinity of $\theta = T/T_c$ equal to 0.95; 0.975 (NH_3, CO_2, and CF[illegible]Cl intersect $\theta = 1$ at an angle less than $\pi/2$). For H_2O and C[illegible]H[illegible], however, a minimum appears at $\theta = 0.975$ and $\theta = 0.95$, respectively.

It is known that, as the critical region is approached, the accuracy of measurement of vapor pressures diminishes, and, also, the presence of noncondensable gases leads to considerable error in the pressure measurements. However, in the case of water, the first measurements of vapor pressure using precise thermometry and the pressure balance were reported in 1910 by Holmorn and Baymann. These measurements were followed by three independent series of measurements obtained during the International Steam Program sponsored by The American Society of Mechanical

Engineers. Authors compute the α vs. θ curve for the observations reported in each of the four investigations and find general accord in the α vs. θ relation with a minimum at $\theta = 97.5$, or about 16 deg below the critical temperature. Measurements were carried out with accuracy to within a degree of the critical temperature, and there seems little reason to suspect the minimum to be the result of an undetected trend away from true values of the pressure-temperature relationship. The Sidney Young measurements were reported 40 years ago, and a decision on the course of α for these substances should be based upon new measurements using modern thermometry and pressure measuring devices. With this in mind, authors show that the criterion, $\alpha = 0$, provides a law or means for restricting the forms of empirical equations which may be used to represent the vapor pressure relationship to the critical temperature. For example, the much used form deducible by representing the latent heat by the equation $L = L_0 + \int (C_{P1} - C_{P2})\,dT = L_0 + a_1T + a_2T^2 + \dots$ is shown by authors to be incapable of representing data as the critical region is approached. A modification is proposed, however, which is based on assuming $L = L_0 + a_1T + a_7T^7$ for substitution in the Clapeyron equation and integrating on the assumption that volume of the liquid phase may be omitted.

Frederick G. Keyes, USA

1777. **English, Robert E., and Wachtl, William W., Charts of thermodynamic properties of air and combustion products from 300 to 3500° R,** *Nat. adv. Comm. Aero. tech. Note* 2071, 108 pp., Apr. 1950.

Nomographs similar to those previously prepared for dry air by A. Amorosi [*Bur. Ships, Navy Dept., Res. Memo.* no. 6–44, Dec. 1944] are presented for mixtures of air, water vapor, and products of combustion of hydrocarbon fuels for temperatures from 300 to 3500 R, hydrogen-carbon ratios from 0.10 to 0.20, and fuel-air ratio from zero to stoichiometric. Properties presented are enthalpy and $\int (c_p/T)dT$. Gas imperfection and dissociation are neglected. Numerical examples illustrate application to gas-turbine computations.

Serge Gratch, USA

1778. **Andersen, J. R., Some new values of the second enthalpy coefficient for dry air,** *Trans. Amer. Soc. mech. Engrs.* **72**, 6, 759–765, Aug. 1950.

The second enthalpy coefficient is the temperature function β in the equation $h = h(\text{at } p = 0) + \beta p + \gamma p^2 + \dots$ where h denotes enthalpy per unit mass and p the pressure. Thus, it is the slope of an isotherm at zero pressure on an $h - p$ diagram. It is directly related to temperature function B in the equation of state $pv = pv\,(\text{at } p = 0) + Bp + Cp^2 + \dots$ It is, therefore, an important quantity in fixing the relations between properties of a fluid at low pressures.

Author measured β for air by means of isothermal steady-flow expansion process at 0, 10, and 30 C. He formulated these results by assuming that relation between β and T (temperature) would correspond to that indicated by Lennard-Jones intermolecular force potential $E(r) = 4\epsilon[(r_0/r)^{12} - (r_0/r)^6]$ where r denotes separation of a pair of molecules. The minimum potential ϵ and the separation corresponding to zero potential r_0 were considered to be constants for all temperatures considered and were evaluated from the experimental values of β. Resulting formulation gave values of β in good accord with other measurements covering a somewhat greater range of temperature. The formulation was revised to improve agreement with measurements by Eucken, Clusius, and Berger.

A temperature function B corresponding to this function β is stated and compared with experimental values. The means of obtaining a constant C in the expression for B, although not clearly stated, probably involves adjustment to empirical data on p-v-T relation.

Comparison of β formulation with several older ones indicates that new values are less above 25 C and greater below 0 C, a discrepancy attributed by author principally to errors in Roebuck's Joule-Thomson data which were employed by earlier investigators.

A discussion by F. G. Keyes traces history of isothermal expansion experiment and discusses validity of assumed Lennard-Jones force potential.

Joseph H. Keenan, USA

1779. **Johnston, H. L., and White, David, Pressure-volume-temperature relationships of gaseous normal hydrogen from its boiling point to room temperature and from 0–200 atmospheres,** *Trans. Amer. Soc. mech. Engrs.* **72**, 6, 785–787, Aug. 1950.

Paper presents a summary of accurate measurements on data of state for normal hydrogen gathered over a period of years by a group of investigators in The Ohio State University Cryogenic Laboratories. Results cover a wide range of temperature and pressure.

From authors' summary

1780. **Bowden, A. T., Gas turbines for industrial purposes,** *Engineering* **170**, 4415, 214–216, Sept. 1950.

Article covers a general discussion of applications and limitations of gas turbines for industrial purposes. Temperature entropy diagrams are used to compare the gas-turbine and steam-turbine cycles. Such factors as reactions between metals and gases, quality of fuels, compact and efficient heat exchangers, and maximum permissible gas temperatures and their importance in the development of industrial gas turbines are discussed in some detail. Some of the design and economical factors which are influencing, and will likely continue to influence the development of gas turbines for industrial purposes are presented.

George A. Hawkins, USA

1781. **Roy, Maurice, Theoretical investigations on the efficiency and the conditions for the realization of jet engines,** *Nat. adv. Comm. Aero. tech. Memo.* 1259, transl., 232 pp., June 1950.

Paper is translation of "Recherches théoriques sur le rendement et les conditions de réalisation des systèmes motopropulseurs à réaction," Publications Scientifiques et Techniques du Ministère de l'Air, Service des Recherches de l'Aéronautique, 1930. It is a comprehensive mathematical treatise of the efficiencies of rocket motors and jet motors with and without thrust augmentation. After an introductory section on definitions of efficiencies—thermal, mechanical, over-all—the work is divided into two main parts. Part I deals with rockets, powder and liquid-fuel propellants; part II with the jet propeller. Three appendixes deal with steady flow of viscous gases, viscous fluids in nozzles, and thrust augmenters. Paper gives complete details of the deduction of the various formulas, well illustrated with numerical tables. It is a thesis rather than a research paper.

R. C. Knight, England

1782. **Richardson, F. D., and Jeffes, J. H. E., The thermodynamic background of iron and steelmaking processes,** *J. Iron Steel Inst. Lond.* **163**, part 4, 397–420, Dec. 1949.

A survey is made of thermodynamic data available for the main compounds of importance in ironmaking. Free energy and heat changes of the most important reactions have been calculated up to 2000 C, and manner in which they vary with temperature is shown by means of suitable diagrams on which probable limits of accuracy are indicated. Results are discussed and are applied to processes occurring in the blast furnace, in terms of oxygen potential. Deductions concern effects of temperature,

pressure, and concentration on interlocking of many simultaneously occurring reactions. From authors' summary

1783. Bryan, W. L., and Quaint, G. W., Heat transfer coefficients in horizontal tube evaporators, *Refrig. Engng.* **59,** 1, 67–72, Jan. 1951.

Refrigeration evaporator designers need extensive heat-transfer and pressure-drop data for various fluids. Authors have made experimental detemination of same for Freon-11 evaporating under forced circulation in a 10-ft, $^3/_8$-in. copper tube. Flow rates, heat inputs, pressure losses, and tube temperatures were established by direct measurement; refrigerant temperatures from pressure data. Readings were also taken with a brass spring inserted as a turbulence promoter.

Thermal data are presented as local and average coefficients and correlated in terms of Nusselt's forced convection equation and weight of fluid vaporized per unit length of tube. They are related to pressure drops by plot of h/k vs. $\Delta P/L/D$ with Freon-12 and water data.

Reviewer believes that results constitute a dependable contribution to fund of data on two-phase fluid flow and to evaporator design information. Value of latter is limited by conditions of incomplete evaporation and high pressure loss and by lack of generalization. Use of rate of vaporization instead of temperature difference as parameter is inconclusively justified.

William H. Roberts, Jr., USA

1784. Kramers, H. A., On the behaviour of a gas near a wall (in Italian), *Nuovo Cim.* (9) **6,** *Suppl.* 2 (Conv. inter. statist Mecc.), 297–304, 1949.

The slip velocity u_0 of gas at the wall when there is a shear velocity gradient du/dz normal to the wall was computed approximately by J. C. Maxwell [*Trans. roy. Soc. Lond. Ser. A*, **170,** 231–256, 1879] assuming (1) that the molecular velocity distribution function at the wall for incident molecules is not influenced by the re-emitted molecules from the wall, and (2) that there is a fraction f of the impinging molecules re-emitted perfectly diffuse, whereas the rest $1 - f$ undergo specular reflection. For isothermal flow, slip number k, defined in terms of the mean free path l as $u_0 = kldu/dz$, is then $(2 - f)/f$. Experimental results gave $k \sim 1.2$. Thus $f < 1$. Present author believes that the most reasonable assumption is to take $f = 1$, and the deviation of k from unity must be sought from a more exact solution of the molecular velocity distribution function near the wall. Starting from the Boltzmann integrodifferential equation for the molecular velocity distribution function, author deduces a linear homogeneous integral equation for the deviation from the Maxwellian distribution function, and k is then an eigenvalue of the problem. It is evident that k must be dependent upon the manner of interaction between molecules. No solution is given. However, author shows that k must be larger than unity by qualitative arguments. H. S. Tsien, USA

1785. Noyes, Robert N., Prandtl-Meyer flow for a diatomic gas of variable specific heat, *Nat. adv. Comm. Aero. tech. Note* 2125, 22 pp., June 1950.

An analysis is presented of the Prandtl-Meyer expansion round a corner for a diatomic gas, in cases in which high temperature or pressure ratio cause appreciable variation of specific heat. Effect of the specific heat lag on the flow is assumed negligible and the expression representing the vibrational energy of the molecules is taken from the classical kinetic theory of gases.

It is shown, as would be expected, that a close approximation to the variable specific heat solution for moderate expansions is obtained by using a constant specific heat ratio appropriate to the initial temperature. Using this approximation, corrections to certain of the flow variables appear to change sign, and reviewer wonders whether this is due to computational difficulties.

G. H. Lean, USA

1786. Damköhler, Gerhard, Isentropic phase changes in dissociating gases and the method of sound dispersion for the investigation of homogeneous gas reactions with very high speed, *Nat. adv. Comm. Aero. tech. Memo.* **1268,** 1269, 58 pp., 41 pp., Sept. 1950.

Translation from articles in *Z. Elektrochem.* **48,** 2, 3, 1942.

1787. Sato, Mizuho, On the depression of freezing point of water due to the internal stress accompanying the transformation, *J. sci. Res. Inst. Tokyo,* **44,** 1209, 6–9, Sept. 1949.

1788. Koenig, F., On the thermodynamic relation between surface tension and curvature, *J. chem. Phys.* **18,** 4, 449–459, Apr. 1950.

In a mathematical study, author extends Tolman's deductions [AMR **3,** Rev. 2679] relating surface tension to radius of curvature for one component system to multicomponent systems. Discussion is restricted to the case of contact of two immiscible fluid phases in stable equilibrium, no component in the surface not found in at least one of the phases, with the surface curved in any way compatible with equilibrium. Using Gibbs' adsorption theorem and the Gibbs-Duhem equations, two expressions are obtained which are jointly generalized solutions of the problem. From these, special equations applicable to experimental situations commonly encountered in surface tension measurements are derived. In the special case where the interface is spherical, the two expressions reduce to a single equation which is a generalization of Tolman's. R. E. Treybal, USA

1789. Szell, K., On the fluctuation of energy of gases in the Bose-Fermi quantum statistics, *J. chem. Phys.* 18, 5, 636–638, May 1950.

Fluctuation in energy of a monatomic gas is calculated on the basis of the Bose-Fermi statistics, both by using the partition functions and using the entropy equation.

Keith J. Laidler, USA

1790. Baer, H., Pressure drop and change of state in long pipe lines for gas and steam (in German), *Forsch. Geb. Ing.-Wes.* 16, 3, 79–84, 1949–1950.

Change in pressure, volume, and speed of gases in long horizontal pipes with constant diameter is calculated. A graphical method is given for steam, which permits, through use of the Mollier diagram, calculation of the heat radiation and takes account of the friction coefficient as a function of the Reynolds number along the conduit.

Translation from author's summary

1791. Dow, Willard M., The uniform distribution of a fluid flowing through a perforated pipe, *J. appl. Mech.* 17, 4, 431–438, Dec. 1950.

Paper deals with problem of designing a pipe manifold which discharges fluid through ports in the wall over a considerable length. It is desired to attain the same flow rate through the ports over whole length. Application is made to the gas "pipe burner," in which it is found that important increases in capacity can be obtained. Solution is achieved, for constant diameter ports, by determining that variation in cross-sectional area along the pipe which will result in a constant static pressure. Author

has obtained experimental verification of the theory in case of the gas burner over a certain range of the variables. Solution should not be expected to hold for very high flow rates, however, as author has omitted from his basic equation (1) terms which account for energy transported out of the pipe by fluid passing through the ports. Reviewer would expect these terms to be important, especially near the dead end.

The obvious alternative solution of varying the port areas in a constant diameter pipe is not discussed. When flow rate is such that greater discharge tends to occur near dead end, then decreasing port areas appropriately in that region will restore uniform discharge, and vice versa when dead-end discharge tends to be low.

Bernard Etkin, Canada

Heat and Mass Transfer

(See also Revs. 1430, 1597, 1684, 1743, 1768, 1783)

1792. **Jacob, Max, Rose, R. L., and Spielman, Maurice, Heat transfer from an air jet to a plane plate with entrainment of water vapor from the environment,** *Trans. Amer. Soc. mech. Engrs.* **72**, 6, 859–867, Aug. 1950.

Paper presents some experimental results obtained by discharging hot air from a continuous slot parallel to a plane surface. A method for predicting the distribution of surface temperature, relations for the temperature and vapor pressure in the jet, a common correlation relating heat transfer and mass transfer, considering also the entrainment of environment air by the jet, and a numerical example are given.

From authors' summary

1793. **Vodicka, V., Heat conduction in a cylinder segment** (in German), *Schweiz. Arch.* **16**, 10, 304–308, Oct. 1950.

Author solves two-dimensional steady-state problem of infinitely long cylindrical segment for arbitrary temperature distributions θ_1, θ_2, θ_3 on three surfaces $\phi = 0, \phi = \alpha$, and $r = R$, respectively, where α is the subtended angle and R radius of the segment. Transformation $u + iv = (i\pi/\alpha) \log (R/x + iy)$ maps the sector on semi-infinite strip $0 \leq u \leq \pi, v \geq 0$. After applying Laplace transform, differential equation and boundary conditions can be separated into two parts. One part has obvious closed solution; the other part is solved by a Green's function. Transform solution, which is sum of these two solutions, contains $\partial\theta(u,0)/\partial v$. Author shows how this can be expressed in terms of the transforms L_1 and L_2 of θ_1 and θ_2, respectively. The solution is simplified by expanding the Green's function and an expression containing L_1 and L_2 as two meromorphic functions, each with first-order poles. Solution consists of equation for temperature θ which can be computed directly when θ_1, θ_2, and θ_3 are specified.

This is an excellent illustration of several well-known mathematical methods adroitly combined to solve a very general problem.

H. A. Lang, USA

1794. **Danckwerts, P. V., Unsteady-state diffusion or heat-conduction with moving boundary,** *Trans. Faraday Soc.* **46**, part 9, 701–712, Sept. 1950.

A general method of solution is described for a class of problems in unsteady-state linear heat conduction or diffusion, which involve two phases or regions separated by a moving plane interface. Following examples are used to illustrate the method: Absorption by a liquid of a single component from a mixture of gases; tarnishing reactions; condensation of a vapor at surface of a cold liquid or on a cooled surface; gas reacting at solid surface to form gaseous product; solution of a gas in a liquid, followed by reaction with solute; and progressive freezing of a liquid.

From author's summary by M. C. Huppert, USA

1795. **Sestini, Giorgio, On the heat condition in a thin plate bounded by two concentric circles** (in Italian), *Atti Semin. mat. fis. Univ. Modena* **3**, 125–137, 1949.

Let r,θ be polar coordinates, $f(r)$, $\phi_1(t)$, $\phi_2(t)$ given continuous functions, and a and b constants. The following boundary-value problem for a circular disk is treated using the Laplace transformation:

$$T_{rr} + r^{-1}T_r - T_t = aT - b,\ 0 < r_1 < r < r_2;$$

$$t > 0,\ \lim_{t \to 0} T(r,t) = f(r),\ T(r_1,t) = \phi_1(t),\ T(r_2,t) = \phi_2(t),\ t > 0$$

The Laplace transformation is also used to construct a Green's function for the above boundary-value problem. Without further assumptions on T, author's uniqueness "proof" must be considered questionable.

Courtesy of Mathematical Reviews F. G. Dressel, USA

1796. **Hutchinson, F. W., Experimental evaluation of human shape factors with respect to floor areas,** *Trans. Amer. Soc. mech. Engrs.* **72**, 6, 889–891, Aug. 1950.

Results are presented for the shape factor of an average standing person with respect to energy in the form of heat received from a floor area.

From author's summary

1797. **McMahon, H. O., Bowen, R. J., and Bleyle, G. A., Jr., A perforated-plate heat exchanger,** *Trans. Amer. Soc. mech. Engrs.* **72**, 5, 623–632, July 1950.

Experimentally determined effects of hole diameter, plate thickness, and plate separation on the heat transfer and flow resistance for a perforated-plate, gas to gas, heat exchanger are presented. Reynolds number based on hole diameter ranged from 800 to 4300. Film coefficients of heat transfer, based on inside surface area of hole, were as high as 237 Btu per (hr)(sq ft)(deg F). One-eighth-in. diam holes on $^3/_{16}$-in. equilateral triangular spacing and 0.0833-in. diam holes on $^1/_8$-in. equilateral triangular spacing were used. The plates were $^1/_{16}$ and $^1/_8$ in. thick. Neoprene gaskets separating the plates in the direction of flow were varied in thickness from 0.013 in. to 0.123 in.

Byron E. Short, USA

1798. **Eckert, E. R. G., and Low, George M., Temperature distribution in internally heated walls of heat exchangers composed of noncircular flow passages,** *Nat. adv. Comm. Aero. tech. Note* 2257, 39 pp., Jan. 1951.

To estimate temperature distribution within a solid mass of constant high thermal conductivity, within which heat is generated at constant volumetrically uniform rate, while heat is removed by coolant of unity Prandtl number having constant entering temperature, heat capacity, viscosity, and low thermal conductivity flowing turbulently through long straight passages which traverse the mass, Poisson's equation for heat balance within solid can be integrated numerically on assumption that at interface, product of solid's thermal conductivity and internal temperature gradient equals product of film heat-transfer coefficient and difference between surface and bulk coolant temperatures. Knowing that average film coefficient for noncircular passage is same as for circular of same hydraulic diameter, and taking variation of film coefficient around periphery as proportional to variation of wall shear around periphery reported by Nikuradse, calculations were made for rectangular and triangular cross sections, assuming temperature in solid varies only in flow

direction, and for rectangular passages with temperature varying normal to flow direction also. Benjamin Miller, USA

1799. McAdams, W. H., Kennel, W. E., and Addoms, J. N., Heat transfer to superheated steam at high pressures, *Trans. Amer. Soc. mech. Engrs.* **72**, 4, 421–428, May 1950.

Superheated steam 115–3500 psia, 430–1000 F, flowing between concentric vertical tubes 0.252- and 0.382-in. diam, was heated electrically (through inner tube) and the temperature rise was measured by thermocouples. Local coefficients of heat transfer h calculated from 212 test points were plotted against Re number (7,000–40,000) using data for mean film temperature. The final equation $Nu = 0.0214\ [1 + 2.3/(L/D_e)]Pr^{1/3}Re^{0.8}$ with fade-away function for L/D represents the results with maximum error of 17%. A valuable paper touching interesting questions.

Otakar Mastovsky, Czechoslovakia

1800. Andronikashvili, E. L., On the question of heat transfer in helium II (in Russian), *Zh. exp. teor. Fiz.* **19**, 6, 535–542, June 1949.

1801. McAdams, William H., Heat transfer (in Dutch), *Ingenieur* **62**, 47, ch. 81-ch. 90, Nov. 1950.

1802. Elser, Karl, Friction temperature fields in turbulent boundary layers (in German), *Mitt. Inst. Thermodyn. Verbrennungsmotor. E.T.H. Zurich*, no. 8, 88 pp., 1949.

Author derives analytical expressions for temperature distribution caused by friction in a compressible fluid flowing past an insulated wall. Empirical velocity-distribution data are employed to formulate heat-momentum transfer analogy for classical cases of flow in a circular cylinder and over flat plates; for the former case, buffer zone is neglected and flow régime is divided into boundary layer and turbulent core.

Considering velocity fluctuations as following a Maxwellian distribution leads to value of 1.09 for ratio of turbulent exchange quantities, eddy conductivity, and viscosity. This compares favorably with value of 1.1 obtained experimentally by Reichardt.

Comparison is made between friction factor defined on the basis of wall shear stress and that based on pressure drop. For incompressible fluids the two are identical: for compressible flow the former may be estimated from the von Kármán-Nikuradse formula while the latter is a function of Mach number and Reynolds number. Results of tests on air illustrate the comparison.

A. Shaffer, USA

1803. Thompson, A. S., Flow of heated gases, *J. appl. Mech.* **17**, 1, 91–98, Mar. 1950.

A graphical method is given for calculation of the interdependence of pressure, temperature, Mach number, and area along a passage through which a heated gas flows at high speed. It is assumed that the flow is one-dimensional and steady, that no shock waves are present, and that the gas is perfect. For purpose of developing design charts, the flow along the passage is broken up into steps. Along each step, wall temperature is assumed to be constant and process is assumed to be composed of an isentropic process taking place in zero length and a constant Mach number process covering the entire step length; constant Mach number is a fictitious one obtained by calculating a weighted average from the actual Mach number variation across step. Analysis closes with an examination of the limitations on the flow imposed by the second law of thermodynamics.

Charts for use in calculations are presented together with an illustrative example. The charts are based on assumption that the Reynolds analogy between heat transfer and skin friction is valid. A discussion is included of the effect on calculated pressure and area ratios of keeping the ratio of specific heats constant and equal to 1.4.

Reviewer refers those interested in a more general treatment of the flow of gases in passages to a paper mentioned by author in a footnote [Shapiro, A. H., and Hawthorne, W. R., "The mechanics and thermodynamics of steady one-dimensional gas flow," AMR 1, Rev. 332].

Neal Tetervin, USA

1804. Hoge, Harold J., Thermal properties of gases, *Nat. Bur. Stands. Table* 19.50, July 1950.

Vapor pressure of argon. Ed.

1805. Arthur, James S., Specific heats of MgO, TiO_2, and ZrO_2 at high temperature, *J. appl. Phys.* **21**, 8, 723–733, Aug. 1950.

1806. Winston, Gerald, and Backer, Stanley, Measurement of the thermal transmission of textile fabrics, *Amer. Soc. Test. Mat. Bull.* **162**, 62–67, Dec. 1949.

Various techniques for measuring thermal-transmission characteristics of textile fabrics were studied to develop a standard test for evaluation of this property. All techniques employed were based upon the principle of determining energy required to maintain a body at a constant temperature when covered by the test specimen. Of all fabric properties, thickness evidenced closest relationship with thermal insulation, indicating that, for the present, thickness values should suffice to predict fabric performance on laboratory equipment designed to evaluate thermal transmission of fabrics under still-air conditions.

From authors' summary

1807. Langstroth, G. O., and Hart, K. H., The specific heat of Napalm-gasoline gels, *Canad. J. Res. Sec. A*, **27**, 151–155, July 1949.

Heat capacities of gasoline, Napalm, and various Napalm-gasoline gels have been measured in the range −50 C to +50 C by method of mixtures, using an ordinary type of Richard's adiabatic calorimeter. From authors' summary

1808. Cummings, G. H., and West, A. S., Heat transfer data for kettles with jackets and coils, *Indust. Engng. Chem.* **42**, 11, 2303–2317, Nov. 1950.

An extension of the general method of correlating heat-transfer data for forced convection in an agitated vessel is presented. Six liquids of widely differing physical and thermal properties (water, toluene, isopropyl alcohol, ethylene glycol, glycerol, and white mineral oil) were used in the experimental work. In addition, some data are presented for two liquid-solid systems, and for two liquid-liquid systems. Two different types of agitators, a retreating-blade turbine and a 45° pitched-blade turbine, were used. All of the data obtained in this work together with previously reported data are correlated.

From authors' summary

1809. Mendelssohn, K., and Olsen, J. L., Heat flow in superconductive alloys, *Proc. phys. Soc. Lond. Sec. A*, **63**, part 10, 370 A, 1182–1183, Oct. 1950.

Heat conductivity was measured in normal and superconductive state of pure lead alloyed with 0.02, 0.1, 0.2, 0.5% bismuth. Both states can be produced at low enough temperatures (below 7 K) by a variable magnetic field. It was found that for 0.02 and 0.1% Bi the heat conductivity in the superconductive state is always appreciably lower than in normal state. For 0.5% Bi

conditions are reversed and at 0.2% Bi crossover occurs, the supercritical conductivity being lower between 5 and 7 K, and higher below 5 K than the normal state value. In normal state all heat conductivities decrease with increasing alloy content. In supercritical state some crossover occurs. Explanation is offered by comparison with conditions in liquid helium II.

E. Eckert, USA

1810. **Hatfield, H. Stafford, and Wilkins, F. J., A new heat-flowmeter,** *J. sci. Instrum.* **27**, 1, 1–3, Jan. 1950.

A heat-flowmeter having the form of a small disk of tellurium-silver alloy with copper-gauze coating on its two sides is placed with its plane at right angles to flow of heat to be measured, creating a small proportionate difference of temperature between the two sides. Output of the disk is 2.0μV for a flow of 1-Btu/ft² hr. Internal resistance including leads is 1Ω. Disk is connected by fine wires through a terminal block some distance away to a galvanometer which registers the thermo-emf due to difference in temperature of the two tellurium-copper junctions. Disk is calibrated by placing it in a measured heat flow through a number of layers of plush. Experiments carried out in a room-size steel-walled chamber with one or more of its walls heated gave an approximately correct figure for the heat imparted to injected air current, the volume and change of temperature of which were measured.

From authors' summary

Acoustics

(See also Revs. 1470, 1786)

1811. **Kinsler, Lawrence E., and Frey, Austin R., Fundamentals of acoustics,** New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; Oct. 1950, vii + 516 pp. $6.

First nine chapters of book present an analysis of the vibrations of solid bodies and the propagation of sound in fluids. Remaining seven chapters are each a self-contained description of a particular application. On the whole, the fundamentals of acoustics are developed with great clarity and insight, making for easy reading. Book should find a place as a text and can be recommended as a reference for the nonspecialist. Treatment is almost entirely nonhistorical; in most cases, investigators associated with advances in the subject are not even named. Only the chapter on psychoacoustics is free from this defect. Typography is very good; illustrations are clear and well drawn, although in many cases the legends are inadequate.

Martin Greenspan USA

1812. **Harris, Cyril M., On the acoustics of coupled rooms,** *J. acoust. Soc. Amer.* **22**, 5, 572–578, Sept. 1950.

Problem is simplified to that of change in resonant frequency and damping of a normal mode of a rectangular room with an adjustable partition parallel to wave front, and dividing room unequally. Method of Feshbach [*Phys. Rev.* **65**, p. 307, 1944] is applied to cases of large and small opening in partition, and calculations agree with experiment. In both cases, resonant frequency increases with size of opening, while damping decreases as size increases for large opening, but increases for small one. Frequency shift varies as square of unperturbed velocity for large opening and as square of unperturbed pressure for small one.

V. Salmon, USA

1813. **Shaw, E. A. G., Attenuation of (1, 0) "transverse" acoustic waves in a rectangular tube,** *J. acoust. Soc. Amer.* **22**, 4, 512–513, July 1950.

A rigorous expression is given for that portion of the attenuation arising from viscosity and heat conduction in the gas near the walls. It is compared to a similar one of Hartig and Lambert, [title source, **22**, p. 42, 1950], and several differences are noted in the analytical and experimental work. New relation agrees with values measured in dry nitrogen to 1%. Effect of absorbing tube surfaces is now being investigated.

V. Salmon, USA

1814. **Igarashi, Juichi, The determination of sound absorption coefficient,** *J. phys. Soc. Japan* **5**, 4, 249–253, July-Aug. 1950.

Author describes measurement of absorption coefficients of various materials by two methods: (1) By measuring reverberation time in a small concrete chamber containing distributed samples of these materials, and (2) by measuring decay time of sound introduced at one end of a tube having the test material at opposite end. A modified approximate formula for reverberation time for the nonuniform field, as in the chamber, is introduced. Absorption coefficients determined by first method were greater than those of the second. Author prefers second method for determining relationship of absorption coefficient as a function of sound frequency for ease and accuracy of measurement.

Morris S. Macovsky, USA

1815. **Nyborg, W. L., Rudnick, I., and Schilling, H. K., Experiments on acoustic absorption in sand and soil,** *J. acoust. Soc. Amer.* **22**, 4, 422–425, July 1950.

Acoustic absorption of sand and soil with varying amounts of water added has been investigated in the frequency region from 10 to 100 kc. The unsaturated media were found to have an attenuation of from 2 db/cm to 25 db/cm, the value for any particular sample being correlated with the acoustic flow resistance. In water-saturated media the attenuation depended markedly on the amount of gas present on account of scattering from gas bubbles. Air-free mixtures had a very low attenuation probably approaching that of water.

Warren P. Mason, USA

1816. **Mikhailov, I. G., and Gurevich, S. B., Absorption and velocity of ultrasonic waves in some very viscous liquids and amorphous rigid bodies** (in Russian), *Zh. exp. teor. Phys.* **19**, 3, 193–201, Mar. 1949.

1817. **Hueter, T. F., Ultrasonic velocity dispersion in solid rods,** *J. acoust. Soc. Amer.* **22**, 4, 514–515, July 1950.

Note describes brief theory and apparatus useful in obtaining phase velocity of ultrasonic waves in case of high frequency configurational dispersion in cylindrical solid rods of radial symmetry. Velocity is calculated from knowledge of radii of nodes of axial displacement at one end of rod. Apparatus used to detect these nodes is piezoelectric pick-up utilizing small drop of mercury as coupling between rod and crystal. Mercury ball is constrained to rest on face of crystal at center, as rod end is scanned along a diameter.

Paul Tamarkin, USA

1818. **Baccaredda, Mario, Ultrasonic velocity and adiabatic compressibility in some isomeric benzine bisubstituted derivatives** (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Fis. Mat. Nat.* **6**, 4, 466–471, Apr. 1949.

1819. **Chartier, Charles, Bourot, Joseph, and Noel, Jean, Visualization of vibratory phenomena in a sonor tube** (in French), *C. R. Acad. Sci. Paris* **230**, 26, 2269–2270, June 1950.

Describes an experimental procedure for studying vibrations in a resonant pipe by photographing fine particles of aluminum powder suspended in the gas. Accuracy of method depends on the frequency and the suspension coefficient of the particles. A

theoretical equation is given indicating best results with low frequency and fine powders. Two photographs are included; one with continuous light indicating amplitudes only, and the second one with interrupted light and a rotating mirror giving a dashed sine curve which indicates velocities. W. B. Stiles, USA

1820. Schneider, W. G., and Thiessen, G. J., The velocity of sound in helium at temperatures −78° C to 200° C and pressures up to 70 atmospheres, *Canad. J. Res.* **28,** 5, 509–519, Sept. 1950.

The use of sonic velocity data to find the zero-pressure velocity, the second virial coefficient B, and the specific heat ratio γ, is discussed and illustrated with helium, which has PV isotherms linear in P. A double crystal acoustic interferometer at 600 kc with motor drive and automatic response recorder was employed. Experimental velocities were about 0.1% above theoretical. They were estimated accurate to 0.1%, and their slope vs. pressure to 1%. B was obtained as a function of T by 2- and 3-power equations, which agreed well when the pressure coefficient of γ was computed from them.

It is concluded that the sonic velocity method is comparable in accuracy to the usual compressibility methods for obtaining equation of state data, has no advantage, is experimentally somewhat more difficult, but is a valuable check. C. F. Bonilla, USA

1821. Morkovin, M. V., A note on the velocity of sound, *J. aero. Sci.* **17,** 3, p. 180, Mar. 1950.

1822. Sette, Daniele, Propagation velocity of ultrasonics in liquid mixtures (in Italian), *Ric. sci.* **19,** 11–12, 1338–1379, Nov.-Dec. 1949.

1823. Kobrynski, M., On a method of calculating the noise spectrum of airplanes (in French), *Rech. aéro.*, no. 11, 37–45, Sept.-Oct. 1949.

1824. Merbt, H., and Billing, H., The propeller as rotating sound source (in German), *Z. angew. Math. Mech.* **29,** 10, 301–311, Oct. 1949.

Problem is to calculate the field of sound of rotating airscrews, which is dependent on the geometrical data, number of revolutions, angle of incidence, and advance ratio of the screw. The sound fields of a source and of a doublet moving around a circular or helicoidal path are determined. With them, the airscrew is represented (sources corresponding to its thickness, doublets to its incidence). Theoretical results are compared with Ernsthausen's measurements on model screws.

Gino Moretti, Argentina

1825. Beranek, Leo L., Elwell, Fred S., Roberts, John P., and Taylor, C. Fayette, Experiments in external noise reduction of light airplanes, *Nat. adv. Comm. Aero. tech. Note* 2079, 121 pp., May 1950.

Present work is part of a program to find practicable ways of reducing noise of light aircraft in order to make them less objectionable to persons on the ground. In general, it was demonstrated that significant reduction in the external noise can be achieved without basic changes in airplane structure and without serious sacrifices in performance. Noise levels with the best combinations tested were, in the opinion of observers, probably lower than is essential to eliminate most public objections to such aircraft on account of their noise characteristics.

Results confirm previous work in that both engine muffling and a reduction of propeller tip speed were found necessary to achieve satisfactory noise reduction. Increasing the number of propeller blades was also beneficial. Data are presented in form of plots giving sound intensity on ground as a function of distance to airplane, for various test and flight conditions.

A. Regier, USA

Ballistics, Detonics (Explosions)

(See also Rev. 1674)

1826. Blanc, P., Exterior ballistics of rockets in the vacuum and with constant gravity field (in French), *Mémor. Artill. fr.* **24,** 1, 7–158, 1950.

This long discussion, dealing with a situation far removed from reality, seeks to present the essentials of a self-contained theory sharing certain aspects in common with the ballistics of projectiles fired from cannon, on the one hand, and with aeronautics on the other hand. Author introduces from the start suitable dimensionless variables which must play a leading role in any theory. Using many graphs, tables, and equations, he considers a sequence of well-chosen more or less general cases, starting with rectilinear motion in a vacuum and without gravity. Author's many qualitative inferences may seem of questionable interest to one concerned with actual trajectories in resisting air. For small rocket weapons whose fuel combustion is completed near the launching tube, the discussion may provide little utility. Tables, in part to six significant figures, occupy about 30 pages. Author is not concerned with physical chemistry of the propellant. Definition of N, on page 11, is erroneous and may confuse the reader. Presumably this exposition may serve as introductory chapters of an elaborate theoretical treatise on rocket motion. Albert A. Bennett, USA

1827. Brändli, Hans, Theory of the multiple shot (Theorie des mehrfach Schusses), Basel, Verlag Birkhäuser, 1950, 198 pp. Sw. fr. 28.50.

Applications of probability theory to the problem of destroying airplanes by firing a number of projectiles at them are developed. This field of applied probability theory is seldom treated in conventional texts. Necessary leads for hitting moving targets are first derived by vector methods, and general requirements for hitting are discussed. Probability of destroying an airplane is then developed for single rounds when the projectiles have contact fuses, time fuses, combination time-contact fuses, and proximity fuses. Book assumes, in general, trivariate Gaussian distribution of the round-to-round errors, superimposed on systematic displacements of the target from the means of the random errors.

The mathematics and geometrical relations for fragmenting projectiles bursting near targets are developed. Fragment effects are shown in fixed and moving coordinate systems. Methods are given for determining the relative effectiveness of time-fused and contact-fused projectiles, based on the concept of "vulnerability circle" in the target. Optimum relationship between random and systematic errors for maximizing probability are given.

Problem of destroying a single target with many rounds is considered. Fuses are proximity, contact, and combination time- and-contact. Methods are given for determining the required number of rounds for various probabilities of destruction.

Theory is extended to include fire from a number of guns separated from each other, and the geometric arrangement of the guns and its effect on kill probabilities is examined. Firing of many rounds against many targets is considered, and optimum conditions discussed.

Finally, varieties of barrage fire and methods for computing associated probabilities are described. An appendix discusses a particular type of fire-control system.

Book is devoted almost entirely to the mathematical aspect of the problem and is not concerned with target or weapon characteristics. Although emphasis is specifically on antiaircraft projectiles fired against airplanes, the same mathematical development can be applied to a variety of weapons.

Herbert K. Weiss, USA

1828. **Friedman, Bernard, Theory of under-water explosion bubbles,** *Comm. pure appl. Math.* **3**, 2, 177–199, June 1950.

Interaction of detonation products with water may be divided into the delivery of a short sharp shock in which the gases lose half their energy, followed by a relatively slow expansion of the gas bubble, which overexpands and oscillates. Paper gives a full treatment of the second stage, taking water to be incompressible and pv^γ = const for the gas, where $\gamma \sim 1.3$. Potential theory is used for the water flow and boundaries are allowed for by image methods.

Method gives good predictions of period of oscillation and motion of the C.G. of the bubble, but poor estimates of pressure at minimum bubble size. An interesting result is that rigid surfaces attract the bubble while free surfaces repel it.

H. H. M. Pike, England

Soil Mechanics, Seepage

(See also Revs. 1632, 1633)

1829. **Buisson, M. M., Admissible loads of deep foundations, theoretical and experimental data** (in French), *Ann. Inst. tech. Bât. Trav. publics* (*N.S.*), no. 145, 91–106, Sept. 1950.

Author summarizes investigations made on bearing capacity of deep foundations, shearing resistance of fine grained soils, penetration resistance of coarse-grained soils, and pile-driving formulas. For the bearing capacity of deep foundations a comparison is made of the work of Caquot and Terzaghi. Author states that results due to Caquot are better than those due to Terzaghi because the latter neglects a term related to lateral frictional resistance of the soil.

The hardness of a soil obtained from an indentation or penetration test is used to measure the frictional resistance of a soil. An empirical expression relating the angle of internal friction to the dry density, specific gravity, and plasticity index is given.

Field measurements of resistance to pile penetration are included. Safety factors as applied to the pile driving formula of Crandall are discussed. Bulb piles are treated.

Elio D'Appolonia, USA

1830. **Mizuno, Takaaki, On the bearing power of soil in a two-dimensional problem,** *Memo. Fac. Engng. Kyushu Univ.* **11**, 2, 33–105, 1949.

See AMR **3**, Rev. 1383.

1831. **Bendel, L., The dynamic tri-axial trials,** *Proc. sec. int. Conf. Soil Mech. Found. Engng.* **3**, 131–139, June 1948.

Author describes a self-constructed apparatus in which triaxial experiments, both static and dynamic, may be executed with variable vertical and horizontal loadings. 100-mm high cylinders are tested with a cross-sectional area of 20 cm². Both pressure and deformation are measured by electric devices, thus yielding very reliable results. Tests described consist of a static loading of 1.5-kg/cm² applied in all directions, followed by a dynamic strain with a slowly increased frequency from 0 to 30.5 cps.

At the beginning, the dynamic-loading sets up a sudden settlement of 2 mm, followed by a slight increase up to the critical frequency (21 cps) when a sudden collapse occurs, but no further settlement follows.

Author distinguishes two kinds of pressure: (1) Plate working (large pressing surface), and (2) pile working (small pressing surface). He sets up formulas of settlement for both static and dynamic loading: (a) Large pressing surface, $s = K \log(\sigma_0 + \sigma_z)/\sigma_0$; (b) small pressing surface, $s = a\sigma^b$. Here σ denotes initial compression value, σ_z denotes upper limit of compression value, K is index of logarithm, a and b indexes of logarithm.

The ratio of settlement under dynamic loading to that under static loading: $S_{dyn} = \alpha S_{stat}$, where the value of α decreases with the increase of σ and increases with the increase of the water content.

Photographs illustrate the difference in movement of soil grains set up by dynamic and static pressures. Author concludes that under small surfaces there is no practical difference in the movement of the grains. Under dynamic influence the pure water does not flow away and the carrying element works more vertically, whereas the static-carrying element works more sideways.

Unfortunately, in the English translation, terms are not always correct and sometimes may be misleading.

Ch. Széchy, Hungary

1832. **Charny, I. A., Method of successive change of stationary states and its applications to the problems of nonstationary filtration of liquids and gases** (in Russian), *Izv. Akad. Nauk SSSR Ser. tekh. Nauk*, no. 3, 323–342, March 1949.

1833. **Perret, William R., Electrical resistivity exploration,** *W'ways Exp. Stat. Bull.*, no. 33, 48 pp., Sept. 1949.

First part of the bulletin is devoted to theoretical and general considerations of electrical resistivity methods; it is not original. Second part illustrates, by a detailed description of an actual exploration program, the application of the information presented in first part.

From author's summary

1834. **Fox, E. N., The mathematical solution for the early stages of consolidation,** *Proc. sec. int. Conf. Soil Mech. Found. Engng.* **1**, Sec. 1d3, 41–42, 1948.

Paper deals with the consolidation of a clay layer with an impermeable face and one free drainage face, subjected to a uniform pressure. Author derives the parabolic form of the degree of consolidation in its early stage from operational calculus, and so can give numerical estimates of the accuracy of this solution. He gives, also, an alternative form of the solution for higher degrees of consolidation.

R. Spronck, Belgium

1835. **Wyllie, M. R. J., and Rose, Walter D., Application of the Kozeny equation to consolidated porous media,** *Nature* 165, 4207, p. 972, June 1950.

Agreement is shown between specific surface areas calculated from pressure on a nonwetting phase required to displace a wetting fluid divided by interfacial tension compared to (2.5 × porosity × permeability)$^{-1/2}F^{-1}$. F is the ratio of electrical resistivity of saturated medium divided by resistivity of fluid. Tests are reported on uniform grain size sandstone and sintered Alundum and Pyrex.

Edward S. Barber, USA

1836. **Kirkham, Don, Potential flow into circumferential openings in drain tubes,** *J. appl. Phys.* 21, 7, 655–660, July 1950.

An approximate expression is found for velocity potential outside drain tubes embedded in porous soil. Author utilizes an approximate solution of the corresponding plane problem to find a simple solution in the case of axially symmetric flow into the drain tube. He then uses method of images to account for pres-

ence of the soil surface. Upper and lower bounds for the flux per unit length of drain tubing are also computed.

John A. Lewis, USA

1837. Luthin, James N., and Gaskell, R. E., Numerical solutions for tile drainage of layered soils, *Trans. Amer. geophys. Un.* **31,** 4, 595–602, Aug. 1950.

Application of relaxation methods to solution of Laplace's equation is developed. Method is illustrated by solving a two-layer problem with a tile drain trenched into the lower layer. Equipotential lines and quantity of flow are shown for permeability ratios from 5 to 100. Comparison with the analytical solution for the problem of a tile drain in a uniform soil above an impermeable layer shows this method to be reasonably accurate.

E. S. Barber, USA

1838. Penrod, E. B., Measurement of the thermal diffusivity of a soil by the use of a heat pump, *J. appl. Phys.* **21,** 5, 425–427, May 1950.

Thermal diffusivity of the particular lean clay studied (120 lb/cu ft, low plasticity, moisture about 16% by dry weight) was determined to be 0.02 sq ft/hr.

C. M. Duke, USA

1839. Brosch, Carlos Dias, Interpretation of humidity tests of cast sands (in Portuguese), *Bolet. Assoc. Brasil Metais.* **6,** 136–150 1950.

Paper summarizes three methods dealing with moisture content of sand. Methods are not new. Relation is shown between humidity and volume, and permeability and resistance are plotted against humidity. Immediate applications of the shown graphics in practical engineering is not likely.

Reviewer's opinion: Subject must be studied in relation to the critical densities of the material, the shearing resistance, and the plasticity. It is not mentioned under which conditions the permeability was measured, so the results cannot be compared with others.

Hendrik Jan Oosterbeek, Brazil

1840. Ruckli, Rob. F. X., Two- and three-dimensional groundwater flow towards the ice-lenses formed in the freezing ground, *Proc. sec. int. Congr. Soil Mech. Found. Engng.* **3,** 282–287, June 1948.

Geophysics, Meteorology, Oceanography

(See also Revs. 1683, 1714)

1841. Godson, Warren L., Generalized criteria for dynamic instability, *J. Meteor.* **7,** 4, 268–278, Aug. 1950.

A solution of the equations of frictionless atmospheric motion employing the "parcel" method is offered in which the external forces are assumed to be linear functions of the space coordinates. Stability of the parcel's motion is shown to depend upon the local gradients of temperature and geostrophic wind. The solution is derived by assuming a constant Coriolis parameter and then simplified by omission of the gravitational stability. Certain cases of instability give rise to a "perturbation divergence" which, it is suggested, may be related to weakening of anticyclones or deepening of cyclones.

L. Machta, USA

1842. Masuda, Y., On the resonance of pressure waves and temperature waves, *J. meteor. Soc. Japan* **28,** 5, 139–148, May 1950.

It is shown that dynamical pressure changes may be replaced by upper pressure waves, under some special assumptions; thus the differential equation for pressure change due to pressure and temperature waves is obtained. After solving this equation under different initial conditions, it is seen that, under a special condition, the resonance of pressure with temperature waves may occur. In the last section, a typical example is given and figures for the changes of pressure distribution are obtained.

From author's summary

1843. Platzman, George W., The motion of barotropic disturbances in the upper troposphere, *Tellus* **1,** 3, 53–64, Aug 1949.

From the data collected for 80° W by Palmen and Newton it is found that absolute vorticity of the atmosphere is consistently greater to the north of the maximum zonal velocity than to the south. A model is chosen which has one constant value north of a given latitude and a smaller value from there to the equator. This zonal circulation has a small irrotational disturbance superimposed which is then described in terms of its projection from the south pole on the tangential plane at the north pole. It is necessary to assume the motion to be horizontal, and the solution is expressed as an infinite series, the terms of which are those solutions which have n waves encircling the earth.

After asserting that a disturbance synthesized by superposition of all harmonics is not systematically dispersed but that its evolution is repeated after a time dependent only upon the magnitude of the vorticity discontinuity, it is found that the rate of propagation of energy depends upon n, so that the energy of a disturbance whose form is periodically repeated is progressively redistributed. This paradox is not explained, and difficulties are increased because author switches indiscriminately from local Cartesian to spherical polar coordinates, and then to projection on the polar plane. The form taken by the equation of continuity in high latitudes appears to have escaped his attention; indeed, the equation is never stated.

The reader's patience is greatly taxed by a superfluity of jargon in the introduction, quantities such as "potential vorticity" being defined, never to be mentioned in later sections. The suffix O, a superfixed star, and an imposed bar are all used with two distinct meanings in this short paper. Having defined a function $E_t(\,, t)$, the function $E_t(-t)$ is used without saying whether a comma or one of the arguments has been omitted. All this makes reading very difficult.

Though impressive at first sight, paper is disappointing in content, but author appears conscious of its shortcomings. He has assumed no vertical motion in a level of the atmosphere where it is a maximum, and a shearing current with no corresponding thermal field. His justification appears to be that "baroclinic processes" are "not yet accessible to theoretical analysis." There is excuse for ignorance of a paper by Eady in this same volume of *Tellus*, but not of the work of Charney as far back as 1947, and both papers have shown that they are indeed accessible. His claim that one of his formulas may have prognostic value is surprising; linear extrapolation from two successive weather charts would be far more accurate.

R. S. Scorer, England

1844. Robitzsch, M., The virtual wind (in German), *Z. Meteor.* **3,** 8/9, 247–251, Aug.-Sept. 1949.

1845. Shiotani, M., Turbulence in the lowest layer of the atmosphere (in Japanese), *J. meteor. Soc. Japan* **28,** 6, 181–187, June 1950.

Structures of wind are evaluated from data of observations carried out both in summer and in winter. Several properties of turbulence are larger in summer than in winter in the air layer more than 10 m above the ground. Also, the relation between

mean velocity and turbulence of wind is obtained. Velocity fluctuations and mixing length increase nearly proportionally to increase of mean velocity, and eddy viscosity increases with square of mean value. Phenomena of transition from laminar flow to turbulent flow are described. u' (fluctuation of velocity along mean flow) is always negatively correlated with T' (fluctuation of air temperature). It was assured that turbulence of air temperature is due mainly to local convection on a clear and calm day, whereas turbulence of wind is almost due to random movement of lumps of air (mixing motion).

From author's summary

1846. **Shiotani, M., and Yamamoto, G., Atmospheric turbulence over the large city-turbulence in the free atmosphere (2nd report),** *Geophys. Mag.* **21,** 2, 134–147, Mar. 1950.

Paper presents a discussion of measurements of wind-velocity fluctuations over a town in heights between 25 and 60 m by use of 5 hot-wire anemometers. Evaluation of mean velocity, intensity and scale of turbulence, spectrum of turbulence and eddy viscosity is carried out according to the methods of Prandtl, Taylor, Hesselberg, and others. Due to unstable weather conditions the results do not seem to be of general validity. Horst Merbt, Sweden

1847. **Davies, D. R., Three-dimensional turbulence and evaporation in the lower atmosphere, I, II,** *Quart. J. Mech. appl. Math.* **3,** part I, 51–73, Mar. 1950.

The problems of evaporation and diffusion of vapor into the earth's turbulent airstream above and down-wind from saturated liquid surfaces have formerly been solved by author for parabolic boundaries, using a mathematical model of diffusion worked out by O. G. Sutton (1934). Aim of present paper is to integrate these differential equations for finite rectangular boundaries, again assuming the airstream to be fully turbulent and directed parallel to longitudinal length of the lake. A first approximation is based on assumption that wind speed and diffusivity are constant, so that the diffusion equation reduces to the common heat-flow problem in two dimensions. Method is then generalized on basis that wind speed and diffusivity, both vertical and lateral, obey simple power laws. A formal solution is given up to the down-wind edge of the rectangular lake in terms of Mathieu functions, whereas down-wind of lake the solution can readily be computed from power series. Application of the formal solution to special cases awaits the complete tabulation of Mathieu functions. Part II gives an evaluation of an empirical power law for variation of the coefficient of lateral diffusivity with height above ground from known results for a continuous point source, such as a single smoke generator producing a cloud of airborne particles of matter. In addition, results of experiments in the field, arranged by the Ministry of Supply, over the down-wind edge of a parabolic strip of short grassland initially contaminated with aniline are communicated and found to agree very closely with assumed power law. Walter Wuest, Germany

1848. **Batchelor, G. K., The application of the similarity theory of turbulence to atmospheric diffusion,** *Quart. J. roy. meteor. Soc.* **76,** 328, 133–146, Apr. 1950.

A. N. Kolmogoroff's similarity hypotheses of small scale structure of turbulent motions are evaluated for application to atmospheric diffusion studies. Hypotheses apply for Reynolds numbers of the turbulent motion above approximately 50,000. Principles hold for eddies whose dimensions are less than 100 m and great as compared to 0.2 cm. Application to diffusion from a fixed source and the relative location of neighboring particles moving with the fluid indicate that it does not apply in the first case but does in the second. Comparisons with experimental data indicate general agreement. It is apparent that considerable care is needed in applying the hypotheses.

P. Donely, USA

1849. **Pasquill, F., The aerodynamic drag of grassland,** *Proc. roy. Soc. Lond. Ser A*, **202,** 1068, 143–153, June 1950.

A detailed account is given on the experimental determination of the drag force on a small area of grassland under atmospheric conditions well measured. The boundary-layer velocity profile seems to follow a universal distribution if the parameters for roughness are properly fitted. An estimation of the Richardson number and of the "eddy viscosity" completes the paper.

Leslie S. G. Kovásznay, USA

1850. **Redfield, Alfred C., The analysis of tidal phenomena in narrow embayments,** *Pap. Phys. Oceanogr. Meteor.* **11,** 4, 36 pp., July 1950.

An extension of the theory of damped tidal waves in closed channels is made to nonuniform channels. This is done by means of assumptions permitting the mathematical model worked out classically for the uniform estuary to be applied in nonuniform cases by an appropriate determination of the parameters from tidal observations. Assumptions in the extension are three: (1) Effect of irregularities in cross section is to distort distribution of phase differences along channel; (2) damping is proportional to phase change in waves, rather than to distance traveled; and (3) damping coefficient appearing in equation for the superposed oncoming and reflected waves is a constant. The model is shown to apply with a fair degree of approximation to two nearly uniform channels (Bay of Fundy and Long Island Sound) and one whose departure from uniformity is great (Juan de Fuca Strait). The clever "ansatz" used by another thus permits an amazingly simple model to relate major features of tidal systems (time of high water, relative heights along the channel, relative time of maximum current, etc.), which at the outset are extremely complex in appearance. Joanne Starr Malkus, USA

1851. **Rees, M. R., The equilibrium distribution of the long-period tides over an ocean covering the northern hemisphere,** *Quart. J. Mech. appl. Math.* **3,** part I, 80–88, Mar. 1950.

Paper is the first attempt to study effects of both continents and the mutual attraction of the elevated water upon the equilibrium tide by means of a method due to Brillouin. Main result is that the elevation is about 1.126 times that obtained by neglecting the effect of mutual attraction. Henry Stommel, USA

1852. **Van Straaten, L. M. J. U., Periodic patterns of rippled and smooth areas on water surfaces, induced by wind action,** *Proc. K. Ned. Akad. Wet.* **53,** 8, 1217–1227, 1950.

Two types of such pattern are described, one for the first time. Both were observed in very shallow water (1 to 10 cm), and involve oily contamination of the surface. Quantitative description of examples, some excellent photographs, and results of simple qualitative experiments are included. Philip Rudnick, USA

1853. **Wiegel, Robert L., Experimental study of surface waves in shoaling water,** *Trans. Amer. geophys. Un.*, **31,** 3, 377–385, June 1950.

Experiments were made to compare the wave profile in shoaling water with trochoidal shape required by the irrotational theory of Stokes, and to see how shape was maintained as depth increased. Until water became relatively shallow, Rayleigh's assumption that waves behave locally as if depth were constant seemed fairly well justified.

Measurements of the wave profile were made by wire electrodes.

The difficulties associated with this method are not discussed and the only reference is to an unpublished paper; one is left, therefore, uncertain as to the reliability of the results. Results are presented in a series of diagrams many of which are so small as to be a strain on the eyes, and from which no clear picture of the conclusions can be obtained. A mass of tiny triangles, crosses, squares, and circles are grouped around the curves given by Stokes theory, and the symbol z and suffixes m, o, and *ave* appear on the diagrams unexplained in text of legends. On the whole too many results are given and not enough discussion of the essentials. Term "wave steepness" is used for what one usually calls height/length, although it is previously noted how the faces of the waves become "steep," and the troughs flattened, in the shallower water.

The most interesting result is that mean height of the water surface was higher on the beach than at the wave machine. This is because there is a forward mass transport (in accordance with Stokes' theory) so that a head of water is required on the beach because the compensating back flow has to overcome bottom friction; and yet no Reynolds number is discussed nor experiments made in smaller or larger tanks.

One finishes the second or third reading of the paper not at all sure that anything significant has been demonstrated.

H. S. Scorer, England

1854. Arthur, Robert S., Refraction of shallow water waves: The combined effect of currents and underwater topography, *Trans. Amer. geophys. Un.* **31**, 4, 549–552, Aug. 1950.

Application of Fermat's principle to long wave; the rate of change of wave direction is numerically equal to variation of shear along the crest in a scalar field composed of relative wave speed and current component in a direction normal to crest; direction of wave advance turns toward direction of negative shear along crest. As an example, a solution is obtained for wave propagation through an analytical model of a "rip" current which compares favorably qualitatively to wave patterns observed in rips.

Henry Stommel, USA

1855. Spar, Jerome, On the theory of annual pressure variations, *J. Meteor.* **7**, 3, 167–180, June 1950.

A solution employing spherical harmonics is found from the linearized hydrodynamic equations which provide the annual (July minus January) pressure change from the annual ground temperature change. Theory follows that of Jeffreys by integrating the equations in the vertical, but improves upon earlier work by using the adiabatic law for the physical equation. An attempt to introduce friction forces appears to be unsuccessful. Final picture based on first eight terms of the series solution fits the broader features of observed annual pressure changes quite well. Paper does not treat causes of the temperature changes. Reviewer feels that the favorable results of this theory confirm the validity of the hydrodynamic equations as applied to large scale aspects of atmospheric motions.

L. Machta, USA

1856. Wippermann, Friedrich, Maxima air pressure contrasts and wind velocities as function of the earth rotation (in German), *Met. Rdsch.* **2**, 9/10, 257–258, Sept.-Oct. 1949.

1857. Evjen, Sigurd, On the use of areas of rising and falling pressure in the free atmosphere, *Tellus* **2**, 3, 212–221, Aug. 1950.

Author discusses, in a qualitative manner, application of methods in upper air chart preparation and forecasting developed by Petterssen and Priestley. Such application is concerned primarily with prediction of isallobaric changes (isallobars are lines drawn through equal pressure tendencies) at the earth's surface from consideration of inflow and outflow of air at the 700 and 500 millibar levels in the free atmosphere. Through such considerations it appears possible to predict regions of future large surface isallobaric gradients, whereas such predictions are often not possible from indications on surface charts alone.

Warren W. Berning, USA

1858. Reuter, H., On the theory of heat economy of radiation permeable media (in German), *Tellus* **1**, 3, 6–14, Aug. 1949.

In view of fact that large areas of the earth's surface are temporarily or permanently covered by a medium pervious to radiation, i.e., water, snow, or ice, attention is directed to that special feature of heat economy of these media having to do with radiation balance and heat transfer. Primarily, the purpose of theory is to find an explanation for observed distribution of temperature within these media. A first attempt to solve this problem was previously made by author in a study of heat economy of snow cover. Essential basis of present as well as the earlier treatment lies in assumption that snow, ice, and water, while pervious to insolation, are nevertheless impervious to outgoing radiation. Paper treats only simple case of constant incoming penetrating radiation and at same time a constant effective outgoing radiation from surface of medium. One of the principal results is that temperature maximum produced during heating process does not appear at surface, but rather at some depth below. Although the case of water is not treated in this report, results obtained for ice may be applied, with slight modification, to case of calm pure water in view of use of same coefficients of absorption for both water and ice.

From author's summary

1859. Pasquill, F., Some further considerations of the measurement and indirect evaluation of natural evaporation, *Quart. J. roy. meteor. Soc.* **76**, 329, 287–301, July 1950.

Measurements are reported for evaporation from clayland pasture with grass of moderate length. Simultaneous records are given of water loss from evaporimeters and profiles of wind velocity, temperature, and humidity in the lower two meters of the atmosphere. Evaporation rates calculated from wind velocity and humidity data agree on the average quite well with measured evaporation rates. Author suggests use of temperature, humidity, and wind profiles as a means of measuring evaporation losses from fields.

Myron Tribus, USA

1860. Chebotarev, N. P., Evaporation from the soil surface at ground water level lower than the surface (in Russian), *Doklady Akad. Nauk SSSR* **71**, 2, 257–276, Mar. 1950.

1861. Bernhard, R. K., Vibrograph for unidirectional displacement, velocity, acceleration and jerk, *Proc. Soc. exp. Stress Anal.* **8**, 1, 157–172, 1950.

Author describes a vibrograph developed with the primary objective of being an instrument useful for microseismic investigations, i.e., vibrations in the vicinity of the source of vibration. Mechanical system used is a double pendulum connected in tandem by a coupling leaf such that torsional motion is not possible. Crossed leaf springs are used as pivots to minimize friction and play.

Instruments built and described are a horizontal-gage linear spring system with a natural frequency of 0.35 cps and half-aperiodic damping (dashpot), and a nonlinear undamped vertical gage with natural frequency of 0.50 cps. In both cases the frequency obtained is below that calculated by about 10%. Much higher frequencies are possible by changing the spring-mass system.

A differential transformer using a-c or d-c excitation is used to

measure the position or velocity respectively of the pickup arm relative to the case. Recording is by means of standard amplifiers and sensitive galvanometer oscillograph. Provision is included for static and dynamic calibration.

An appendix on theory adequately discusses an accelerometer. Unfortunately, the author dismisses phase lag of a damped seismometer as being impossible. It is perhaps not well known there is invariably phase distortion in the output of a damped seismic velocity or displacement gage. This distortion only becomes negligible at frequencies very much higher than the natural frequency of the instrument. Of some interest is the discussion of the undamped nonlinear system which is used in the vertical gage.

George R. Carlson, USA

Lubrication; Bearings; Wear

1862. **Weber, C., On the hydrodynamic lubrication theory of journal bearings** (in German), *Z. angew. Math. Mech.* **30**, 4, 112–120, June 1950.

Paper constitutes one of many attempts to arrive at workable solution of Reynolds differential equation for pressure distribution in journal and slider bearings of finite width, by representing the solution approximately as the product of a function f of the coordinate in the direction of main flow (sliding motion) and of a function g of the coordinate in the direction of side flow (side leakage). A new feature is that the problem is here solved by variational principles so as to find the optimum approximation. Function f is given in closed form, but function g is to be computed by a numerical method which is illustrated by an example with width/diameter ratio of 1.62, and with a relative eccentricity of the journal of 0.5.

Expression (3) is dimensionally incorrect; factor s (radial clearance) should be squared, and the expressions on page 120 for bearing load P and its components P_1 and P_2 should be altered accordingly.

H. Blok, Holland

1863. **Kahlert, W., The influence of the inertia forces in the hydrodynamic theory of lubrication materials** (correction) (in German), *Ing.-Arch.* **17**, 3, p. 264, 1949.

See AMR **4**, Rev. 1417.

1864. **Wilcock, D. F., Turbulence in high-speed journal bearings,** *Trans. Amer. Soc. mech. Engrs.* **72**, 6, 825–834, Aug. 1950.

Measurements of the operating characteristics of journal bearings up to high surface speeds have revealed abnormalities beyond a certain critical value. A rapid increase in bearing torque, power loss, and oil-film temperature occurs as speed is increased beyond the critical value, while oil flow decreases below normal. These phenomena are attributed to onset of instability or turbulence in the bearing-oil film with an accompanying increase in energy absorption within the film. Results are compared with theory of Taylor [*Trans. roy. Soc. Lond. Ser. A*, **223**, p. 289, 1923] on stability of a fluid contained between eccentric rotating cylinders. The experiments were conducted in a newly designed test apparatus capable of driving large bearings at high speeds, maximum speed obtained being 30,800 fpm.

John T. Burwell, Jr., USA

1865. **Nowell, Louis A., Investigation of bearing materials under various degrees of lubrication in the low-speed range,** *Amer. Soc. Test. Mat. Bull.*, no. 168, 47–53, Sept. 1950.

Paper describes apparatus, procedure, and results of experimental investigation of partial (30° and 60°) bearings of 2-in. diam at speeds from 25 to 350 rpm and loads from 40 to 900 psi. Lubricant was Navy symbol 2190-T having viscosity of 499.0 SUS at 100 F. Bearing material was Navy grade 2 tin-base babbitt and special care was used in controlling its cleanliness in pouring. Author employs modified Amsler test machine. Measurements were made of: friction torque, bearing temperature (from which viscosities are calculated), and oil-film thickness (to determine transition from fluid to boundary lubrication). Good agreement is obtained between transition points as determined by electronic means and by friction measurements; somewhat better agreement is obtained with assumed viscosity than with calculated viscosity. Results, when plotted using assumed viscosity of 160 centipoises, gave nearly same minimum f vs. $(ZN)/P$ values as when viscosities were calculated from bearing temperatures (measured near trailing edge). While stated objective is to determine frictional properties of bearing materials under varying degrees of lubrication from fluid lubrication to boundary lubrication and then to absence of lubrication and author concludes that evidence in report indicates Amsler machine is well suited to this purpose, reviewer questions whether author can thus generalize on basis of data presented.

Reviewer believes paper is more a study of hydrodynamic lubrication than of bearing materials since no data are presented for either conditions of extreme boundary lubrication or of absence of lubrication.

E. E. Bisson, USA

1866. **Simons, E. M., The hydrodynamic lubrication of cyclically loaded bearings,** *Trans. Amer. Soc. mech. Engrs.* **72**, 6, 805–816, Aug. 1950.

Results of an experimental investigation of lubricating-film thicknesses in a journal bearing subjected to various loading conditions are presented and compared with theoretical predictions. General agreement with theory was found, with two important exceptions: (1) Sustained whirling or periodic orbital motion of shaft center in the unloaded or constantly loaded bearing was absent. Only a transient whirling could be obtained which occurred under no load at slightly *less* than one half the shaft speed. (2) Under a rotating constant load or a sinusoidal alternating load applied at half the frequency of shaft rotation, the film thickness remained finite. Under certain steady load conditions, the position of the shaft center showed evidence of rupture of the oil film.

John T. Burwell, Jr., USA

1867. **Salama, M. E., The effect of macro-roughness on the performance of parallel thrust bearings,** *Instn. mech. Engrs., Appl. Mech.* (W.E.P. no. 59), 149–158, Proc. 1950.

Paper presents analysis demonstrating that surface roughness can provide a series of Reynolds-type oil wedges in a parallel thrust bearing with no side leakage. Suggests that this explains why some parallel thrust bearings can support loads as high as those for the tilting pad type. Analysis involves integration of Reynolds' equation to obtain pressure distribution over one wave length of sinusoidal moving surface, with assumed boundary conditions. The expression for load capacity, which is obtained by summing the pressures under all the waves, shows that capacity increases with wave length of surface irregularities. Also, for a given minimum oil-film thickness, there is an optimum wave amplitude for maximum capacity. Expression for coefficient of friction is obtained from integration of shear equation over the film, and plotted against a dimensionless parameter, which corresponds to Sommerfeld number. Author's experiments with controlled wavy surfaces correlate well, in general, with theoretical predictions.

Review feels this careful work demonstrates that relatively long-wave-length surface irregularities increase load capacity of parallel thrust bearing. This aspect was not considered in earlier papers by Fogg [*Instn. mech. Engrs. Proc.* **155**, 49, 1946] and

Shaw [AMR 1, Rev. 176]. However, reviewer believes that wave length and amplitude of macroroughness needed to produce significant improvement in load capacity are far greater than those found in any unintentional periodic irregularities produced by typical machining operations.

Eugene M. Simons, USA

1868. Parker, R. C., Farnworth, W., and Milne, R., The variation of the coefficient of static friction with the rate of application of the tangential force, *Instn. mech. Engrs., Appl. Mech.* (W.E.P. no. 59), 179–184, Proc. 1950.

This investigation was undertaken because of its possible relation to the occasional failure of safety clutches to slip at high overloads. Coefficient of static friction was measured for a number of different materials under impact conditions. An analysis of the conditions of impact, derived from collision between solid bodies and from a suitable hydraulic system, showed the latter to be the more convenient and equally effective method of applying tangential force. Method adopted consisted of feeding oil at a constant rate into an oil cylinder, the piston of which constituted the striking head. The friction members were so arranged that the struck member was in the form of a bar that was clamped between two massive plates. The normal force was applied hydraulically and the tangential force was measured by wire-resistance strain gages.

Static friction was measured for rates of application of the axial force up to 10^3 tons per sec. Under dry conditions friction increased with rate of applied force, an effect more pronounced between a nonmetal and a metal than between metals. Lubrication with oil appeared to diminish the variation of friction with rate of application of force for all combinations of materials tested. Magnitude of this effect was sufficient to contribute significantly to the type of failure mentioned above.

From authors' summary by M. E. Merchant, USA

Marine Engineering Problems

(See also Revs. 1530, 1685)

1869. Hill, J. G., Comparative cavitation tests, *David W. Taylor Mod. Basin Rep.* 732, 2 pp., Sept. 1950.

A propeller design tested in several water tunnels indicated differences in thrust and torque coefficient curves. An 8-in. propeller was tested at Massachusetts Institute of Technology; 8-, 12-, and 16-in. ones at Taylor Model Basin; and an 18-in. one at Wageningen.

J. M. Robertson, USA

1870. Balhan, J., and Van Manen, J. D., The design of cavitation—free ship screws (in Dutch), *Schip en Werf* **17:** 2, 3, 4, 14; 22–31, 44–52, 66–76, 322; Jan. 20, Feb. 3, Feb. 17, July 7, 1950.

Article surveys results obtained by Lerbs in transforming the Betz condition for minimum energy loss of propellers in a homogeneous flow to a minimum condition for ship plus propeller in its variable wake, and also the combination of elements of circulation theory used by Lerbs for practical design of cavitation-free wake-adapted ship's screws. Authors present a variation on Lerbs' method by making use of airfoil inner sections and give two complete numerical calculations of screws thus designed. Their results of cavitation and propulsion tests behind a ship model are discussed. They compare favorably with systematic series screws. Table 3 on p. 68 contains errors, which have been corrected later on p. 322. Authors propose a modified condition for minimum energy loss, which, however, has not yet been tested in practice.

L. Troost, Holland

1871. Hill, J. G., The design of propellers, *Soc. nav. Arch. mar. Engrs.* no. 1 (adv. copy), 27 pp., Nov. 1949.

1872. Emerson, A., Experimental work on merchant ship models during the war, *Trans. N. E. Cst. Instn. Engrs. Shipb.* 64, 6, 289–332, Apr. 1948.

Descriptions of hulls and results of model resistance and propulsion tests on single-screw cargo vessels and coasters are given in first part of paper. Designs represent a considerable part of the range of single-screw ships. Results are used to illustrate an analysis of factors affecting smooth-water performance and, in particular, of choice between "hollow" and "convex" bows. Individual results are briefly discussed.

From author's summary

1873. Pieterse, G. W., A simple approximation method for the calculation of large changes of trim (in Dutch), *Schip en Werf* 16, 24, 511–515, Nov. 1949.

1874. McDonald, J. H., and Macnaught, D. F., Investigation of carbo distribution in tank vessels, *Soc. nav. Arch. mar. Engrs.*, no. 9 (adv. copy), 23 pp., Nov. 1949.

1875. Schiff, Leonard I., and Gimprich, Marvin, Automatic steering of ships by proportional control, *Trans. Inst. mar. Engrs.* 61, 189–208, Nov. 1949.

In a previous study, a criterion for dynamic stability of an unsteered ship, denoted by P_1, has been set up. Present paper deals with directional stability of a steered ship actuated by a proportional control of the form $\delta = -\gamma\theta - \sigma\dot{\theta}$ (δ rudder angle, θ heading angle, $\gamma > 0$, $\sigma > 0$) measured by the quantity q (real part of characteristic exponent that has smallest negative real part for a stable ship). The independent variable throughout the investigation is $s = (V/L)t$, where V is speed, L length of ship, t time. Investigation is based essentially on discussion of a quartic as characteristic equation of a system of differential equations (motion, control, and rudder lag). Besides, Nyquist's stability criterion is used, which proves to be advantageous when rudder lag is constant, although it does not lead to a computation of path. Steady disturbances and impulse disturbances are dealt with. General discussion is illustrated by ample numerical examples for three ships with different dynamic stabilities.

Following important results are arrived at: (1) Increased directional stability q generally accompanies p for the same control mechanism. (2) q should be probably < -0.5, i.e., $|q| > 0.5$. (3) A rate control is desirable for all but the most stable ships. (4) Curves for maximum allowable steering lag have been computed. Most critical situations are expected to arise with small fast ships or bodies. (5) No limitations need be placed on the hydrodynamic parameters of a normal ship in order that it be capable of satisfactory automatic steering.

G. Weinblum, USA

INDEX OF AUTHORS REFERRED TO IN THIS ISSUE

(NUMBERS USED ARE SERIAL NUMBERS OF REVIEWS)